PERIODIC STABILIZATION OF A 1-DOF HOPPING ROBOT ON NONLINEAR COMPLIANT SURFACE

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Abstract: This paper deals with the problem of characterizing and stabilizing periodic orbits of a 1-DOF hopping robot moving over a nonlinear compliant surface. It is shown, through implicit calculations of the Poincare map, that globally stable periodic motions are possible via the injection of nonlinear damping.

Keywords: periodic stabilization, walking and hopping robots.

1. INTRODUCTION

The system considered is the classic simple point mass bouncing on a surface or equivalently, an idealized 1-dof hopping robot. The objective of this work is to introduce compliance in the robot/ground interface in order to control the interaction. The model can be seen as a rigid robot bouncing on a compliant surface or a compliant robot bouncing on a rigid surface.

The current work stems from our efforts in the modeling and control of legged robots. While modeling a legged robot it is important to recall its principal differences with a conventional manipulator arm, the differences which are of fundamental importance but are sometimes overlooked and, in our opinion, not sufficiently emphasized. While a conventional manipulator arm is permanently rigidly fixed to the ground, a legged robot is not. More importantly the "constraint" between the robot foot and the ground is unilateral. Thus given appropriate joint torques the robot is free to leave the ground, which is an impossibility for a conventional arm.

There are two common lines of approach in treating the interaction of such a robot with the ground. The first is a rigid body approach where both the robot and the ground are idealized rigid bodies. A second approach, the one adopted here, is the modeling of the interaction as a compliant phenomenon. In this approach, we typically place a spring/damper module on one of the two interacting objects. The advantage of this model is that it has continuous dynamics governed completely by differential equations and can be used to simulate a variety of interacting surfaces by changing the spring/damper coefficients.

In case of interactions involving non-zero relative velocities between the interacting objects a linear damper produces a discontinuous jump in the interaction force during the first contact, a phenomenon which cannot be physically justified. Also, traditional spring-damper elements, which are active both in extension and compression, cannot correctly model the ground contact of either a bouncing ball or a legged robot since they would generate artificial tension forces pulling the interacting objects towards each other.

In order to rectify the two first problem, we adopt the use of *non-linear* spring-damper elements following (?) and (Marhefka and Orin., April 1996). The authors of both of these articles focus on the performance of the non-linear models in terms of their dynamic behavior. We use the model, in addition, to formulate simple control laws to stabilize a simple one-degree of freedom hopping robot about a desired periodic orbit. The characteristics of the walking robot is preserved here through the restriction of allowing only positive feedback. Given the height of jump altitude, the employed control law tries to bring the robot to the bouncing trajectory. In this paper we present a control law that globally stabilizes the robot to any arbitrary jump altitude. We present two alternative controllers: a dead-beat feedback and an asymptotic one.

Raibert performed a detailed theoretical and practical investigation of the hopping robot (Raibert, 1986) and was capable of establishing a stable vertical hopping motion of the robot. Several theoretical studies, chiefly those from the groups of Koditschek (Koditschek and Buehler, 1991) and Burdick(A.F. *et al.*, 1991)(McCloskey and Burdick., April 1991) followed from this. (Ahmadi and Buehler., April 1995) studied passive dynamic running with a simple hopping robot model similar to the ones mentioned above. This model takes advantage of tuned passive elements, in addition to the actuators to run economically. The proposed controller can stabilize any desired robot speed. (Francois and Samson., April 1994) considered the problem of creating constant-energy gaits of hopping robots.

2. MODELS

The dynamics of the hopping robot may be written as follow:

$$m\dot{v} = -mg + u + F \quad u \geq 0 \tag{1}$$

where m is the masse, v is the velocity of the robot, g is the gravity, $u \in R^+$ is the control input, and F is the contact force.

Nonlinear contact models for compliant surfaces have been introduced in works of (Hunt and Crossley, 1975), and studied further in Marhefka and Orin' 96, to cope with the following drawbacks of the simple linear spring/damper model:

- the model is physically incoherent since interacting bodies may exert tensile forces on another before separation,
- the equivalent restitution coefficient depends upon the masses of the colliding bodies and not on the impact velocities,
- contact force is discontinuous at the moment of impact.

A quite general nonlinear contact model is (Hunt and Crossley, 1975) is:

$$F = -\lambda|x|^n v - k|x|^{n-1}x \tag{2}$$

where F is the contact force, $v = \dot{x}$ is the velocity, x the position, and λ, k are positive constants. The surface $x = 0$ represents the contact surface. The power $n \geq 1$ is used to model different geometries of contact (i.e. a sphere impacting a Hertzian plate ; $n = 2/3$). Two interesting properties have been demonstrated with this model (Marhefka and Orin' 96):

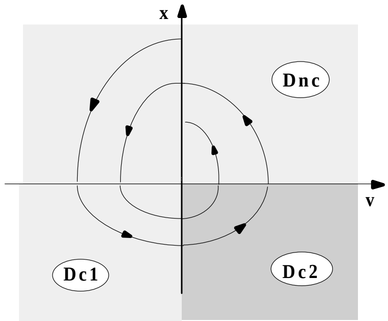

Fig. 1. Natural dissipative motion.

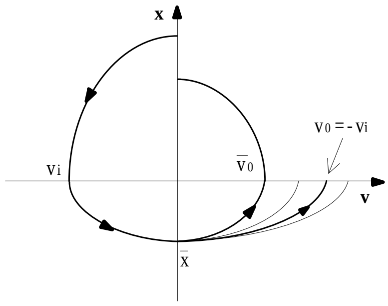

Fig. 2. Accessible set of output velocities by positive damping injection.

(i) consider a body of mass m, in contact with a surface modeled by (1) and (2), i. e.

$$m\dot{v} + \lambda|x|^n v + k|x|^{n-1}x = 0, \qquad x \leq 0$$

then the position x can be solved explicitly as a function of the actual velocity v, the contact velocity v_i, and the contact position x_i

$$x = \phi(x_i, v_i, v) \tag{3}$$

$$\triangleq -\left[\frac{2m(n+1)}{9\alpha^2 k}\left(3\alpha(v - v_i) - 2\ln\left|\frac{2 - 3\alpha v_i}{2 - 3\alpha v}\right|\right) + (-x_i)^{n+1}\right]^{\frac{1}{n+1}} \tag{4}$$

where $\alpha = (2/3k)\lambda$.

(ii) For small αv_i, the equivalent restitution coefficient e,can be approximated as $e \approx 1 - \alpha v_i$.

The complete system can be written as:

$$\dot{v} = \begin{cases} -g & \text{if } x > 0 \\ \dfrac{1}{m}(-\lambda|x|^n v - k|x|^{n-1}x + u) - g & \text{if } x \leq 0 \end{cases} \tag{5}$$

where $x = 0$ describes the contact surface, and $u \in R^+$. Note that the control u, $u \geq 0$ can only apply forces in the upward direction and that no control action is possible during the flight phase. This is a common factor in the walking mechanisms where the degree of freedom between feet and ground is not actuated.

3. INVARIANT PERIODIC ORBITS

The control philosophy underlined in this paper consists first in the characterization, via an inner-feedback loop, of an invariant periodic orbit, and then to design, through an outer-feedback loop, a control strategy to stabilize such an orbit.

Since model (5) can only dissipate energy during the contact phases (the map $v \mapsto -F$ is dissipative), and energy is preserved during the free motion phases, it is clear that "natural" periodic orbits (with $u = 0$) cannot exist. This can be seen by writing the following energy-like positive continuous function V

$$V = \begin{cases} mgx + 0.5mv^2 & \text{if } x > 0 \\ mgx + 0.5mv^2 + \dfrac{k}{n+1}|x|^{n+1} & \text{if } x \leq 0 \end{cases} \quad (6)$$

which has its time-derivative evaluated along the solutions (continuous) of (5) given as

$$\dot{V} = \begin{cases} 0 & \text{if } x > 0 \\ -\lambda |x|^n v^2 & \text{if } x \leq 0 \end{cases} \quad (7)$$

It is convenient to split the state-space into three disjoint subsets in R^2,

$$D_{nc} = \{(x, v) : x > 0\} \quad (8)$$
$$D_{c1} = \{(x, v) : x \leq 0, v < 0\} \quad (9)$$
$$D_{c2} = \{(x, v) : x \leq 0, v \geq 0\} \quad (10)$$

This state-space partition is shown in Fig(1)(2), together with the phase diagram of a natural dissipative motion ($u = 0$). In this figure v_0 and v_i stands for the exit velocity and the entrance velocity, respectively. x_d is the desired altitude of the periodic motion. Note that to reach x_d it is necessary to inject energy via $u \geq 0$ during the contact phases. However, this is only possible during the motion in D_{c2} due to the restriction of the positive sign of the control action.

For this we consider that u be given as:

$$u = g + \frac{\bar{\lambda}_2(v_i)}{m}|x|^n v \geq 0 \quad \text{if} \quad (x, v) \in D_{c2} \quad (11)$$

where the first term compensates for the gravity and the second term is a nonlinear damper used to inject the precise amount of energy to ensure that $v_0 = -v_i$. Note that $\bar{\lambda}_2(v_i) \geq 0$ is constant during the contact phase, but it depends on the entrance velocity resulting from the free fall starting at x_d. The nonlinear term in u is homogeneous to the nonlinear damper of the contact surface. Hence the total damper coefficient $\lambda_1(v_i) = \bar{\lambda}_2(v_i) - \lambda$ can be virtually modified within the following bounds: $-\lambda \leq \lambda_2 < \infty$.

With this first inner-loop, the closed-loop equation becomes,

$$\dot{v} = \begin{cases} f_1(x, v) = -g \quad \text{if } (x, v), \in D_{nc} \\ f_2(x, v) = \dfrac{1}{m}(-\lambda|x|^n v - k|x|^{n-1}x) \\ \qquad \text{if } (x, v), \in D_{c1} \\ f_3(x, v) = \dfrac{1}{m}(\lambda_2(v_i)|x|^n v - k|x|^{n-1}x) \\ \qquad \text{if } (x, v), \in D_{c2} \end{cases} \quad (12)$$

Problem 3.1. (Existence of invariant periodic orbits). Let v_d be the desired entrance velocity corresponding to the desired jump altitude, and assume that $x(0) = x_d$. An invariant orbit does exist, if an entrance velocity-dependent constant $\lambda_2(v_i) > 0$, can be found such that the exit velocity v_o be equal to the entrance desired velocity v_d, i.e. $v_o = -v_i = v_d$.

The natural system motion is defined as being a particular case of the above closed-loop equation with $\lambda_2 = -\lambda$, (or equivalent $u = 0$), and is characterized by the explicit map $\phi(\cdot) \; : v \mapsto x$, $\forall (x, v) \in D_{c1} \cup D_{c2}$

$$x = \phi(v_i, v)$$

the exit velocity resulting from this natural dissipative motion, namely $\bar{v}_o$, can be computed from the implicit equation

$$0 = \phi(v_i, \bar{v}_o)$$

which results in $\bar{v}_o < v_i$.

By injecting a positive damping, as described by the last equation (12), or equivalent by changing λ_2 within the allowed range determined above, we can enlarge the set of accessible output velocities $\bar{v}_o$ to the open set $[\bar{v}_o, \infty)$, which, by construction, includes v_i. This property is shown in Figure (2), and it can be demonstrated as follows.

With this controller, the motion in contact is explicitly characterized by the maps $\phi_1 : v \mapsto x$ in D_{c1} and $\phi_2 : v \mapsto x$ in D_{c2}, i.e.,

$$x = \phi_1(v_i, x_i, v) = \phi_1(v_i, 0, v) \quad (13)$$
$$x = \phi_2(\lambda_2, \bar{v}, \bar{x}, v) = \phi_2(\lambda_2, 0, \bar{x}, v) \quad (14)$$

where $\bar{v} = 0, \bar{x}$ are the values of v and x, respectively, when crossing from D_{c1} to D_{c2} at $v = 0$. The contact position is x_i. In spite of the switching from $-\lambda \to \lambda_2$ when crossing from D_{c1} to D_{c2}, the functions $f_2(x, v), f_3(x, v)$, are such that:

$$\lim_{\substack{x \to \bar{x}^- \\ v \to 0^-}} \{f_2(x, v)\} = \lim_{\substack{x \to \bar{x}^+ \\ v \to 0^+}} \{f_3(x, v)\} = -k|\bar{x}|^{n-1}\bar{x}$$

Hence the system solutions as well the maps $\phi_i(\cdot)$ are continuous. We can thus concatenate these maps and write $x = \phi_2 \circ \phi_1$, $\forall (x, v) \in D_{c1} \cup D_{c2}$. Following the above notation we have that $\bar{x} = \phi_1(v_i, 0, v)$, from which we get,

$$x = \phi_2(\lambda_2, 0, \bar{x}, v) \tag{15}$$

$$= \phi_2(\lambda_2, 0, \phi_1(v_i, 0, v), v) \tag{16}$$

$$\triangleq \Phi(\lambda_2, v_i, v) \tag{17}$$

The problem is now to find λ_2, such that it solves the implicit equation (17) evaluated at $x = 0$, $v = -v_i$, i.e.,

$$0 = \Phi(\lambda_2, v_i, -v_i) \tag{18}$$

which is equivalent to solving for $\alpha_2 = (2/3k)\lambda_2$, in the following transcendental equation:

$$a\alpha_2^2 + b\alpha_2 - \ln(2 + b\alpha_2)^2 + 2\ln 2 = 0 \tag{19}$$

with $a = \frac{1}{\alpha^2}\left[3\alpha v_i + \ln\left(\frac{2-3\alpha v_i}{2}\right)^2\right]$, and $b = -3v_i$, $v_i < 0$ $\alpha = (2/3k)\lambda$. By inspection of (19), we can see that for the suitable range of $\lambda_2 > 0$, this function is continuous and it has the unique solution $\lambda_2 = \lambda$. This solution corresponds to a symmetric positive damping injection, where the energy lost during motion in D_{c1} is recovered in D_{c2} by just inverting the sign λ_2. An example of an invariant motion is shown in Fig. (3) with a bold line, where motion is confined to the shown orbit. This nominal orbit will be denoted in the sequel as

$$x = \Phi^*(v) = \Phi(\lambda_2, v_i, v)|_{v_i=-v_d \lambda_2=\lambda_2^*}$$

where λ_2^* is the value of λ_2 that solves for (19) with $v_i = -v_d$. The problem considered in this section is to add a correction term to the nominal control (11) such as to render attractive the invariant orbit $x = \Phi^*(v)$. For this we consider the following structure for $u \geq 0$

$$u = \begin{cases} g - \dfrac{\bar{\lambda}_1(v_i)}{m}|x|^n v \text{ if } (x, v), \in D_{c1} \\ \\ g + \dfrac{\bar{\lambda}_2(v_i)}{m}|x|^n v \text{ if } (x, v), \in D_{c2} \end{cases} \tag{20}$$

where $\bar{\lambda}_1 > 0$, $\bar{\lambda}_2 > 0$. The control u is always positive, since the v is negative in D_{c1} and positive in D_{c2}. However due to this sign restriction on the control input, it is not possible to make the desired orbit $x = \Phi^*(v)$ attractive in the usual sense. Instead, it is possible (as it will be shown in the following subsection), to find $\bar{\lambda}_1$ and $\bar{\lambda}_2$, such that the exit velocity series $\{v_o(k)\}$ converges to desired value v_d, i.e.

$$v_o(k) \to -v_i(k) = v_o(k-1) = v_d$$

This can be performed either in one cycle or asymptotically, as studied next.

3.1 *Dead-beat Control*

With the controller (20), the closed-loop equations now become:

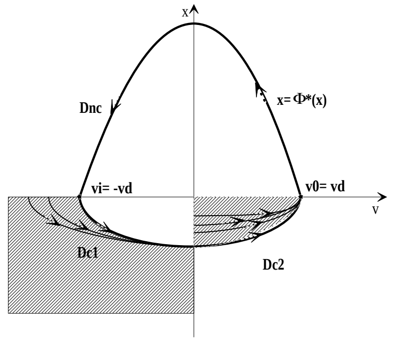

Fig. 3. Accessible sets in the $(x - v)$-plane, and nominal orbit.

$$\dot{v} = \begin{cases} f_1(x, v) = -g \text{ if } (x, v), \in D_{nc} \\ f_2(x, v) = \dfrac{1}{m}(\lambda_1|x|^n v - k|x|^{n-1}x) \\ \qquad \text{if } (x, v), \in D_{c1} \\ f_3(x, v) = \dfrac{1}{m}(\lambda_2|x|^n v - k|x|^{n-1}x) \\ \qquad \text{if } (x, v), \in D_{c2} \end{cases} \tag{21}$$

with $\lambda_1 \in [-\lambda, -\infty)$, $\lambda_2 \in [-\lambda, \infty)$. By modifying these constant with the allowed sets, we can have the following properties:

- (iii) The point $\bar{x} = \Phi^*(0)$ is accessible from any $v_i \in [-v_d, -\infty)$, for motions in D_{c1}.
- (iv) The value of the exit velocity $v_o = v_d$ is accessible from any $x \in (0, -\bar{x}]$, on the axe $v = 0$, for motions in D_{c2}.

Property (iii) results from the ability of dissipating arbitrarily large amounts of energy in D_{c1}, while Property (iv) comes from the fact that energy can be injected in D_{c1}. Then, the shaded area shown in Figure (3) represents the state space domain where energy can be either dissipate or injected throughout positive feedback.

These properties are equivalent to finding λ_1, λ_2 such that the following relationships hold:

$$\bar{x} = \phi_1(\lambda_1(v_i), v_i, 0), \text{ if } v_i \in [-v_d, -\infty), \tag{22}$$

$$0 = \phi_2(\lambda_2(x_c), x_c, v_d), \text{ if } x_c \in (0, \bar{x}], \tag{23}$$

where x_c is the position when crossing the velocity-axis while in contact. In case that the initial contact velocity v_i, is smaller in magnitude than v_d, the value of λ_1 is set to be the natural system damping since there is no need to add more dissipation. Therefore, the maps (22)-(23) are completely defined for any value of v_i, i.e.

$$x_c = \begin{cases} \phi_1(\lambda_1(v_i), v_i, 0) = \bar{x} \text{ if } v_i \in [-v_d, -\infty) \\ \\ \phi_1(\lambda, v_i, 0), \qquad \text{if } v_i \in (-v_d, 0) \end{cases} \tag{24}$$

$$0 = \phi_2(\lambda_2(x_c), x_c, v_d) \text{ if } x_c \in (0, \bar{x}], \tag{25}$$

Since the ϕ_1-map projects all the contact velocities to the bounded set $(0, \bar{x}]$, there is no theoretical need to consider the case where $x_c > \bar{x}$. However, if this happens, due to possible model uncertainties, then λ_2 is set to be equal to $-\lambda$, which will have the effect to bring the system motion towards the interior of the desired orbit $x = \Phi^*(v)$.

As before ϕ_1, ϕ_2 are continuous and can be concatenated to get $x = \phi_2 \circ \phi_1$, $\forall (x, v) \in D_{c1} \cup D_{c2}$, or equivalent,

$$\begin{aligned} x &= \phi_2(\lambda_2, x_c, v) \\ &= \phi_2(\lambda_2, \phi_1(\lambda_1, v_i, v), v) \\ &\triangleq \Phi(\lambda_1, \lambda_2, v_i, v) \end{aligned} \tag{26}$$

Equations (24)-(25) imply that we are able to find λ_1, λ_2 such that it solves the implicit equation (26) evaluated at $x = 0$, $v = -v_d$ for all bounded v_i, i.e.

$$0 = \Phi(\lambda_1, \lambda_2, v_i, v_d) \tag{27}$$

The solution studied in the previous section is a particular case of the above problem. Solution for λ_1, in the Equation (24) as well as the solution for λ_2, in the Equation (25) are involved (except if $x_c = \bar{x}$, in which case the solution is $\lambda_2 = \lambda$), and need to be solved numerically by a root finding algorithm. However, these solutions do exist and are uniquely defined.

We can now consider the Poincaré section $S(x, v) = \{(x, v) | x = 0, v > 0\}$. The Poincaré map between the exit velocity at time instant $t = k-1$, $v_o(k-1)$, and the exit velocity at the time $t = k$, $v_o(k)$ is implicitly given by Equation (26), evaluated as $v_i = -v_o(k-1)$, $v = v_o(k)$ and $x = 0$:

$$\Phi(\lambda_1(v_o(k-1)), \lambda_2(v_o(k-1)), v_o(k-1), v_o(k)) = 0$$

and by continuity of the function $\Phi(\cdot)$, the Poincaré map $\Psi : v_o(k-1) \mapsto v_o(k)$, can be represented as:

$$v_o(k) = \Psi(\lambda_1(v_o(k-1)), \lambda_2(v_o(k-1)), v_o(k-1))$$

Introducing $\tilde{v}(k) = v_o(k) - v_d$, we get:

$$\begin{aligned} &\tilde{v}(k) = \\ &\Psi(\lambda_1(v_o(k-1)), \lambda_2(v_o(k-1)), v_o(k-1)) - v_d \end{aligned} \tag{28}$$

If $\lambda_1(v_o(k-1)), \lambda_2(v_o(k-1))$ satisfy (24) and (25), we have $\Psi(\lambda_1(v_o(k-1)), \lambda_2(v_o(k-1)), v_o(k-1)) - v_d = 0$, which when substituted in the above equation gives

$$\tilde{v}(k) = 0 \tag{29}$$

Hence, dead-beat control is achieved. The following theorem summarizes this result.

Theorem 3.1. Let v_d be the desired velocity needed to reach the desired altitude x_d. Let the control law be defined by

$$u = \begin{cases} g - \dfrac{\bar{\lambda}_1(v_i)}{m} |x|^n v \;\; if \;\; (x, v), \in D_{c1} \\ \\ g + \dfrac{\bar{\lambda}_2(v_i)}{m} |x|^n v \;\; if \;\; (x, v), \in D_{c2} \end{cases} \tag{30}$$

with the contact velocity-dependent constants $\bar{\lambda}_1(v_i)$ and $\bar{\lambda}_2(v_i)$, being such as the relations (24) and (25) hold. Then the system solutions are bounded and the altitude x_d is reached in one cycle, or equivalently the exit velocity $v_o(k) \to v_d$, in one step of finite duration.

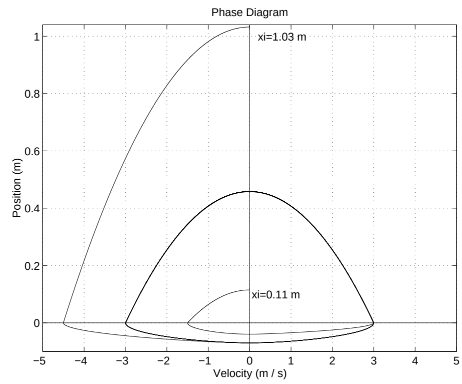

Fig. 4. Phase-plane for the dead-beat control. The parameters for this simulations are: $m = 1kg$; $n = 1$; $k = 1000N/m$; $x_d = 0.46m$; $\alpha = 0.3$

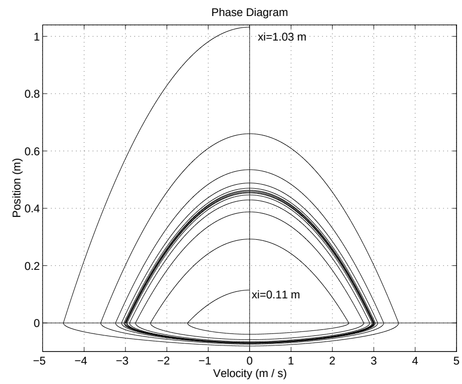

Fig. 5. Phase-plane motion with the asymptotic controller. The parameters for this simulations are: $m = 1kg$; $n = 1$; $k = 1000N/m$; $x_d = 0.46m$; $\sigma = 0.4$; $\alpha = 0.3$

Simulations are shown in Fig (4) starting from two different initial position : $x(0) = 1.03m$ and $x(0) = 0.11m$.

3.2 *Asymptotic stabilization*

We consider now the problem of asymptotic stabilization of equation (28). The motivation for this is twofold: first dead-beat control is known to result in large amplitude control signals, while asymptotic stabilizers necessitate lower control amplitudes. Second, the system robustness will be enhanced.

Assume now that instead of solving for the equation

$$\Phi(\lambda_1(v_o(k-1)), \lambda_2(v_o(k-1)), v_o(k-1), v_o(k)) = 0$$

with $v_o(k)$ set to v_d, we rather solve for,

$$v_o(k) = \sigma \left(v_o(k-1) - v_d\right) + v_d$$

where $|\sigma| < 1$, i.e.

$$\Phi(\lambda_1(v_o(k-1)), \lambda_2(v_o(k-1)), v_o(k-1), \sigma\tilde{v}(k-1) + v_d) = 0$$

which results in

$$\begin{aligned}\tilde{v}(k) &= \Psi(\lambda_1(v_o(k-1)), \lambda_2(v_o(k-1)), v_o(k-1)) - v_d \\ &= \sigma\tilde{v}(k-1) \qquad (31)\end{aligned}$$

since $|\sigma| < 1$, we have that

$$\lim_{k\to\infty} \tilde{v}(k) = 0$$

The following theorem summarizes the result.

Theorem 3.2. Let v_d be the desired velocity needed to reach the desired altitude x_d. Consider the control law 30, with the contact velocity-dependent constants $\bar{\lambda}_1(v_i)$ and $\bar{\lambda}_2(v_i)$, defined now such that the following relationships hold,

$$x_c(k-1) = \begin{cases} \phi_1(\lambda_1(k-1), -v_o(k-1), 0) = \bar{x} \\ \quad if \quad -v_o(k-1) \in [-v_d, -\infty) \\ \\ \phi_1(\lambda, v_o(k-1), 0), \\ \quad if \quad -v_o(k-1) \in (-v_d, 0) \end{cases} \qquad (32)$$

$$\begin{aligned} 0 &= \phi_2(\lambda_2(k-1), x_c(k-1), v_d) \qquad (33) \\ &\quad if \quad x_c(k-1) \in (0, \bar{x}], \end{aligned}$$

with $\bar{x}$ being defined as $\bar{x} = \phi_1(\lambda, -v_d, 0)$. Then the system solutions are bounded and satisfy:

$$\lim_{k\to\infty} v_o(k) = v_d$$

Hence, the jump altitude x_d is reached asymptotically with a convergence velocity defined by σ.

Remark Boundedness of the solution away from the Poincarè section follows from the fact that during contact, the system motion is completely characterized by the relationship $x = \Phi(v_i, v)$. From the continuity and boundedness of the map $\Phi(\cdot)$, and the fact that v_i is bounded (no energy is injected during the flight phase), we have that system solutions cannot escape in finite time. Finally note that for any bounded v_i, Equation (32) projects the system trajectories to $\bar{x}$, from which the map (33) provides a bounded exit velocity, while providing bounded (x, v) trajectories. The above analysis shows that v_d is the unique asymptotically stable equilibrium point for $v_o(k)$.

The phase-plane motion of the simulations are shown in Fig (5) starting from two different initial position: $x(0) = 1.03m$ and $x(0) = 0.11m$, with $\sigma = 0.4$. As mentioned before, the transient magnitude of the control signal can be reduced, as shown the Fig (6).

4. CONCLUSIONS

In this paper we have presented a solution for the problem of characterizing the existence and stabilizing periodic orbits of an actuated 1-DOF hopping robot moving on a nonlinear compliant surface. It was shown that stable periodic motions are possible via the injection of nonlinear damping through nonlinear stabilizing feedback. This gives rise to one-step or asymptotic stabilizers.

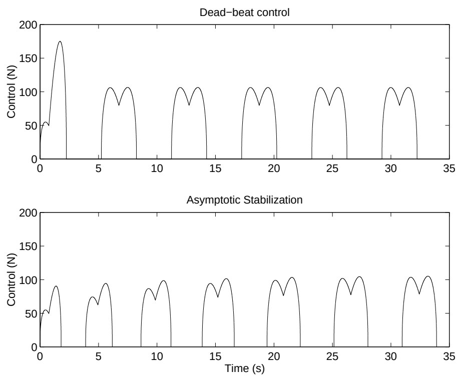

Fig. 6. Time-evolution of the control law. Comparison between the dead-beat and asymptotic strategy.

5. REFERENCES

A.F., Vakakis, Burdick J.W. and Caughey T.K. (1991). An 'interesting' strange attractor in the dynamics of the hopping robot. *Int. J. on Robotics Research* **10**(6), 607–618.

Ahmadi, M. and M. Buehler. (April 1995). A control strategy for stable passive running. *In* Proc. IEEE International Conference on Robotics & Automation*, Nagoya, Japan* pp. 152–157.

Francois, C. and C. Samson. (April 1994). Running with constant energy. *In* Proc. IEEE International Conference on Robotics & Automation*, San Diego* pp. 131–136.

Hunt, K.H. and F.R.E. Crossley (1975). Coefficient of restitution interpreted as damping in vibroimpact. *Transactions of the ASME, Journal of Applied Mechanics* pp. 440–445.

Koditschek, D.E. and M. Buehler (1991). Analysis of a simplified hopping robot. *Int. J. of Rob. Res.* **10**(6), 587–605.

Marhefka, D. and D. Orin. (April 1996). Simulation of contact using a nonlinear damping model. *In* Proc. IEEE International Conference on Robotics & Automation*, Minneapolis* **2**, 1662–1668.

McCloskey, R.T. and J.W. Burdick. (April 1991). An analytical study of simple hopping robots with vertical and forward motion.. *Proc. IEEE Robotics & Automation* **2**(6), 1392–1397.

Raibert, M.H. (1986). *Legged Robots that Balance.* The MIT PRESS. Cambridge Massachusetts, London, England.