# COMPASS GAIT REVISITED

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## Abstract

It has been established that a suitably designed unpowered mechanical biped may "walk" down an inclined plane all by itself and eventually acquire a stable periodic gait. The characteristics of this periodic gait (e.g., velocity, time period) depend on the geometry of the biped and the slope of the plane. The energy to maintain the periodic motion comes from the conversion of the biped's gravitational potential energy as it descends.

In this paper we present an active control scheme for biped robots which mimics the stable periodic motion of a passive biped on an inclined plane. Imagine a biped robot moving on a horizontal plane with a compass gait. If we apply the necessary joint torques in such a way that it induces the precise changes in the states of the robot, its resulting behavior will resemble that of the autonomous motion of a passive robot on a desired slope. We conjecture that the modified behavior of this actively controlled robot will possess the periodicity and the stability of the passive motion.

## 1 Introduction

Nature has invented an extraordinarily vast array of techniques of legged locomotion employing hundreds of legs (as in the centipedes), eight legs (spiders), six legs (insects), four legs (rodents, horses, kangaroos), and two legs (ostriches, human beings). These examples span a large range of speed, payload shape and size, and biomechanical principles. Each form of legged locomotion is efficiently adapted to the lifestyle and environment of the associated creatures. Compared to man-made motive machines on land, which are mainly wheeled machines, legged mobility has several major advantages. For example, a legged system does not need a continuously paved path and its performance is fairly independent of the detailed terrain characteristics [9]. Reasons such as these, despite formidable technical challenge of design and control, has enormously appealed the robotics community into legged locomotion research in the recent time.

One of the most sophisticated forms of legged motion is that of biped locomotion as it is seen in the human beings. From a dynamic systems points of view, human locomotion stands out among other forms of legged locomotion. This is chiefly due to the fact that a significant part of the human walking cycle is not associated with the dynamic equilibrium of the moving body.

The present work, which is still in progress, is aimed at finding simple control methods which would allow a biped robot to walk with a regular gait on a smoothly-varying terrain. This study has its origin in an earlier work of Golliday and Hemami[3] and is based on the interesting more recent work of McGeer[7].

McGeer designed a simple knee-less biped robot and studied its gravity-induced passive motion on an inclined plane. He showed, with the real model and with a linearized mathematical model, that such a biped robot can attain a stable periodic motion. Our objective in this work is to device simple control schemes such that an actively controlled biped robot mimics the motion of its passive counterpart on an inclined plane. Underlying this objective is the conjecture that by behaving similar to a passive system, the controlled biped will enjoy the useful properties (stability, periodicity, and possibly optimality in some sense) inherent in the passive system.

The paper is organized in the following way. In the next section we describe the model that we use in this paper. Section 3 presents the dynamic equations of the model, the equations of transition (relevant during the collision of the robot with the ground), and a linearized version of the non-linear dynamic equations. In Section 4 we investigate the conditions behind the periodic steady-state gait of our model. Section 5 includes a description of the proposed control scheme.

## 2 Adopted model and assumptions

We consider a very simple (academic) model of a biped robot as shown in Figure 1. The characteristics of the model and the associated assumptions are marked below for later reference:

1. *Masses*: All masses are considered point-masses (A1). The lumped mass of each leg is $m$ and the hip mass is $m_H$.

2. *Legs*: The legs are identical (A2). Each leg has a prismatic-jointed knee with a massless (A3) shank. The shank can be telescopically retracted in order to ensure the foot clearance during walk. The leg-length is denoted as $l$ which is divided into $a$ and $b$ as shown in the figure.

3. *Actuators*: A hip torque $u_H$ is always available (A4). An ankle torque, $u_L$ or $u_R$, is also available at the support leg.

4. *Gait*: It consists of the following two stages: a) A "single-support" stage or the "swing" stage in which the system is assumed to behave as a double pendulum (i.e., there is no motion of the contact point) (A5). The biped motion is purely ballistic in this stage.
b) A transition stage between successive left and right support stages. This "double-support" stage or the "stance" stage is assumed to be instantaneous (A6). It is assumed that leg lengths are equal at the transition time (A7). Note that the leg lengths are not equal during the swing stage because of the foot-clearance requirement; however this does not have any consequence on the biped dynamics since the telescopic part of the leg is considered to be massless.

5. *Collision*: The collision of the swing leg with ground is assumed to be inelastic, instantaneous, and without any relative sliding between the contacting surfaces (A8). Collision causes a discontinuous change in the velocity of the biped while keeping its position unchanged.

# 3 The system equations

## 3.1 Dynamics of the single support stage

As stated in the assumptions above, the biped is *always* in the single support stage except during the instantaneous double support stage when the support is transferred from one foot to another. The dynamic equations for this single support stage are rather well-known (see for instance, [8]) and are shown in the Appendix. Since the right leg and left leg of the robot are identical, the equations are similar regardless of the support leg considered. The equations have the following form:

$$\boldsymbol{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \boldsymbol{N}(\boldsymbol{\theta})\dot{\boldsymbol{\theta}}^2 + \boldsymbol{g}(\boldsymbol{\theta}) = \boldsymbol{S}\boldsymbol{u} \quad (1)$$

where $\boldsymbol{M}(\boldsymbol{\theta})$ is the $2 \times 2$ inertia matrix, $\boldsymbol{N}(\boldsymbol{\theta})$ is a $2 \times 2$ matrix with centrifugal coefficients, $\boldsymbol{g}(\boldsymbol{\theta})$ is a $2 \times 1$ vector of gravitational torques and $\boldsymbol{S}$ is a $2 \times 3$ matrix which selects the joint torques. $\boldsymbol{\theta} = [\theta_L \;\; \theta_R]^T$ is the vector of joint angles and $\boldsymbol{u} = [u_L \;\; u_H \;\; u_R]^T$ is the vector of joint torques. The subscripts $L$, $H$, and $R$ denote left foot, hip, and right foot respectively.

## 3.2 Equation of transition

With the assumptions A5 and A7, during transition, simultaneously two things happen : the swing leg touches the ground and the support leg leaves the ground. According to A8 colliding surfaces do not deform and torque is not transmitted about this point. In other words, the angular momentum of the biped about the collision point is conserved before and after the transition. A consequent second condition is that the angular momentum of the non-colliding foot is conserved about the hip joint during the collision. From these two conditions we obtain two equations:

$$\dot{\boldsymbol{\theta}}(T+\varepsilon) = \boldsymbol{H}(\boldsymbol{\theta}(T))\dot{\boldsymbol{\theta}}(T-\varepsilon) \quad (2)$$

where $\dot{\boldsymbol{\theta}}(T-\varepsilon)$ and $\dot{\boldsymbol{\theta}}(T+\varepsilon)$ are the angular velocities just before and after the transition, which takes place at time $t = T$, $\varepsilon$ being a small time interval. Although, for simplicity, transition is assumed to be instantaneous, we should remember that in the actual physical system the hind leg leaves the ground and starts swinging *only after* the front leg has hit the ground. $\boldsymbol{H}(\boldsymbol{\theta}(T))$ is a $2\times2$ matrix (henceforth referred to as $\boldsymbol{H}$ for brevity). At the double support stage, the inter-leg angle $2\alpha$ fully defines the biped geometry (with A2, A3, and A7). In other words, as shown in the appendix, $\boldsymbol{H}$ is a function of $\alpha$ only.

Using A7, we have at the transfer:

$$\theta_L(T) + \theta_R(T) = -2\phi \quad (3)$$

$$\text{and } 2\alpha(T) = \rho(\theta_R(T) - \theta_L(T)) \quad (4)$$

where $\rho = 1$ or $-1$ for left support and right support, respectively.

## 3.3 Introducing the slope in the dynamics

Uptill now $\phi$ appeared only in the transition equation. Now, we would like to explicitly calculate its

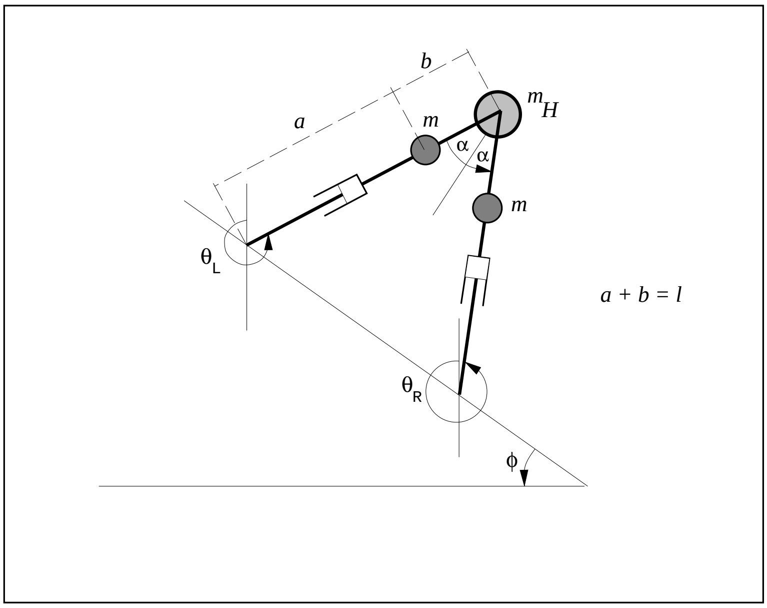


Figure 1: Model of Biped Robot on a Slope. The lumped mass of each leg is $m$ and the hip mass is $m_H$. The leg-length is denoted as $l$ which is divided into $a$ and $b$ as shown in the figure. $\theta_R$ and $\theta_L$ are the angles made by the biped legs with the vertical (counterclockwise positive). The total angle between the legs, the "inter-leg angle", is $2\alpha$. The slope of the ground with respect to the horizontal is denoted by the angle $\phi$.

effect on the dynamical system. This we will do by setting:

$$\boldsymbol{\theta}' = \boldsymbol{\theta} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} \phi \tag{5}$$

The geometrical significance of Equations (5) is that the angles made by the biped with the ground are now measured from the surface normal rather than from the vertical. We observe that

$$\boldsymbol{M}(\boldsymbol{\theta}) = \boldsymbol{M}(\boldsymbol{\theta}') \tag{6}$$

$$\boldsymbol{N}(\boldsymbol{\theta}) = \boldsymbol{N}(\boldsymbol{\theta}'). \tag{7}$$

The gravity term may be expressed as:

$$\boldsymbol{g}(\boldsymbol{\theta}) = g \begin{pmatrix} p \sin \theta_L \\ q \sin \theta_R \end{pmatrix} = g \begin{pmatrix} p \sin(\theta'_L - \phi) \\ q \sin(\theta'_R - \phi) \end{pmatrix} \tag{8}$$

$$\text{where} \begin{cases} p = mb \\ q = -(m(2l - b) + m_H l) \end{cases}$$

for right leg support. Similar results can be found for the single support stage with left leg support.

Equation (1) therefore becomes:

$$\boldsymbol{M}(\boldsymbol{\theta}')\ddot{\boldsymbol{\theta}}' + \boldsymbol{N}(\boldsymbol{\theta}')\dot{\boldsymbol{\theta}}'^2 + \boldsymbol{g}(\boldsymbol{\theta}') = \boldsymbol{S}\boldsymbol{u}. \tag{9}$$

Since the transformation (5) does not affect $\boldsymbol{H}$, we can rewrite the transition equation (2) in new variables as

$$\dot{\boldsymbol{\theta}}'(T + \varepsilon) = \boldsymbol{H}\dot{\boldsymbol{\theta}}'(T - \varepsilon) \tag{10}$$

Similarly (3) and (5) give us the second transition condition

$$\theta'_L(T) + \theta'_R(T) = 0. \tag{11}$$

### 3.4 Linearized models

Although the dynamic model of the biped robot is non-linear we, as a first approach, apply a linearization technique to obtain some insight of the system. Since the non-linear differential equations do not lend themselves to explicit solutions, linearization is very useful. According to ([7]) a linear model is justified for the range of angles used in walking. We, however, have obtained qualitative behavior of the original non-linear system through numerical simulations.

In order to linearize (1), we define the state as $\boldsymbol{x} = [\boldsymbol{\theta} \;\; \dot{\boldsymbol{\theta}}]^T$ and we linearize the dynamics around $\boldsymbol{x} = \boldsymbol{0}$ (i.e., when both legs move around the vertical and the angular velocities are zero).

By denoting $\boldsymbol{M}(\boldsymbol{\theta} = \boldsymbol{0}) = \boldsymbol{M}_0$ and $\boldsymbol{g}(\boldsymbol{\theta} = \boldsymbol{0}) = \boldsymbol{G}_0\boldsymbol{\theta}$, where $\boldsymbol{G}_0 = g \begin{pmatrix} p & 0 \\ 0 & q \end{pmatrix}$ the linearized model may be expressed as :

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \tag{12}$$

where,

$$\boldsymbol{A} = \begin{pmatrix} \boldsymbol{0}_{2\times 2} & \boldsymbol{I}_2 \\ -(\boldsymbol{M}_0)^{-1}\boldsymbol{G}_0 & 0 \end{pmatrix} \tag{13}$$

and,

$$\boldsymbol{B} = \begin{pmatrix} \mathbf{0}_{2\times 3} \\ (\boldsymbol{M}_0)^{-1}\boldsymbol{S} \end{pmatrix} \qquad (14)$$

.

We note that for small angles $\boldsymbol{g}(\boldsymbol{\theta}')$ may be linearized as

$$\boldsymbol{g}(\boldsymbol{\theta}'=0) \simeq \boldsymbol{G}_0\boldsymbol{\theta}' - g\begin{bmatrix} p \\ q \end{bmatrix}\phi \qquad (15)$$

and $\phi$ may be considered as a constant input to the dynamics. This also makes the role of the slope in the system motion explicit. By denoting $\boldsymbol{x} = [\boldsymbol{\theta}' \ \ \dot{\boldsymbol{\theta}}']^T$ and using (15), we obtain the linearized model corresponding to (9):

$$\dot{\boldsymbol{x}}' = \boldsymbol{A}\boldsymbol{x}' + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{D}\phi \qquad (16)$$

with, $\boldsymbol{D} = \begin{bmatrix} 0 \\ 0 \\ g(\boldsymbol{M}_0)^{-1}\begin{bmatrix} p \\ q \end{bmatrix} \end{bmatrix}$.

# 4 Finding a steady-state gait

## 4.1 Introduction to phase portrait

We would like to introduce a typical phase portrait of the biped. This will help the subsequent discussion and will also assist us in visualizing the dynamic state of the biped as it evolves in time. For an introduction to the use of phase portraits in the study of dynamic systems, see [4].

Figure 2 shows the phase portrait of a joint variable ($\theta_R$ selected here). The plane of joint variable vs. joint velocity is selected as the phase plane. For a periodic stationary gait the phase portrait is a cycle. One of the most important characteristics of this phase portrait is the effect of the biped's impact with the ground. During one phase plane cycle the biped undergoes two impacts which correspond to the two transitions – one from the left leg to the right leg and the other from the right leg to the left leg. The effect of these impacts on a joint variable may not be identical. For example, a direct impact of the right foot with the ground might result in a larger velocity shift in $\theta_R$ than that when the left foot hits the ground.

One may start following the curve in Figure 2 from time $t=\varepsilon$, when the right leg just looses contact with the ground (i.e., it becomes the swing leg). At the same time left leg touches the ground (i.e., it becomes the support leg). The corresponding stick diagram shows a black dot on the left foot to imply ground contact. The phase trajectory evolves in the clockwise sense in this diagram as shown by the arrowheads. While crossing the velocity axis (at a positive velocity), the biped is in the vertical configuration. At time $t = T-\varepsilon$, the right leg completes its swing and is about to touch the ground at its maximum joint position. The impact between the right foot and the ground occurs during the short time from $t = T-\varepsilon$ to $t = T+\varepsilon$. At the end of this impact period, the left foot starts its swing and subsequently executes the lower half of the phase plane.

## 4.2 Periodicity condition for the autonomous biped system

Without the presence of the control torques, the solution to the autonomous system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$ between any two time instances $t_1$ and $t_2$ is of the form

$$\boldsymbol{x}(t_2) = e^{\mathrm{A}(t_2-t_1)}\boldsymbol{x}(t_1). \qquad (17)$$

Therefore between the time instances $\varepsilon$ and $T-\varepsilon$ we can write,

$$\boldsymbol{x}(T-\varepsilon) \simeq e^{\mathrm{A}T}\boldsymbol{x}(\varepsilon) \qquad (18)$$

By following McGeer's approach [7], we note that in order for the biped to maintain a steady-state gait, its angular positions and velocities should repeat themselves in a periodic fashion. Moreover, due to the mechanical symmetry of the biped, the left leg and the right leg should alternatively execute identical motions. These conditions may be compactly expressed as:

$$\boldsymbol{x}(T+\varepsilon) = \begin{pmatrix} \boldsymbol{J} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{J} \end{pmatrix}\boldsymbol{x}(\varepsilon) \qquad (19)$$

$$\text{where } \boldsymbol{J} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

By fixing the inter-leg angle at the transition, $2\alpha(T)$, the system is solved in [7] using a Newton method. Let us apply a similar approach, as detailed in [10], to our system. Equations (2) and (18) give:

$$\boldsymbol{x}(T+\varepsilon) = \begin{pmatrix} \boldsymbol{I}_2 & 0 \\ 0 & \boldsymbol{H} \end{pmatrix} e^{\mathrm{A}T}\boldsymbol{x}(\varepsilon). \qquad (20)$$

Equating the right hand sides of the Equations (19) and (20), and taking advantage of the fact that $\boldsymbol{J}^{-1} = \boldsymbol{J}$, we obtain the periodicity equation

$$\boldsymbol{x}(\varepsilon) = \boldsymbol{Q}\boldsymbol{x}(\varepsilon) \qquad (21)$$

with

$$\boldsymbol{Q} = \begin{pmatrix} \boldsymbol{J} & 0 \\ 0 & \boldsymbol{J} \end{pmatrix}\begin{pmatrix} \boldsymbol{I}_2 & 0 \\ 0 & \boldsymbol{H} \end{pmatrix} e^{\mathrm{A}T}. \qquad (22)$$

For Equation (21) to hold, $\boldsymbol{Q}$ should have a unity eigenvalue with $\boldsymbol{x}(\varepsilon)$ as the associated eigenvector. Since $\boldsymbol{H}$ is a function of $\alpha(T)$, which we

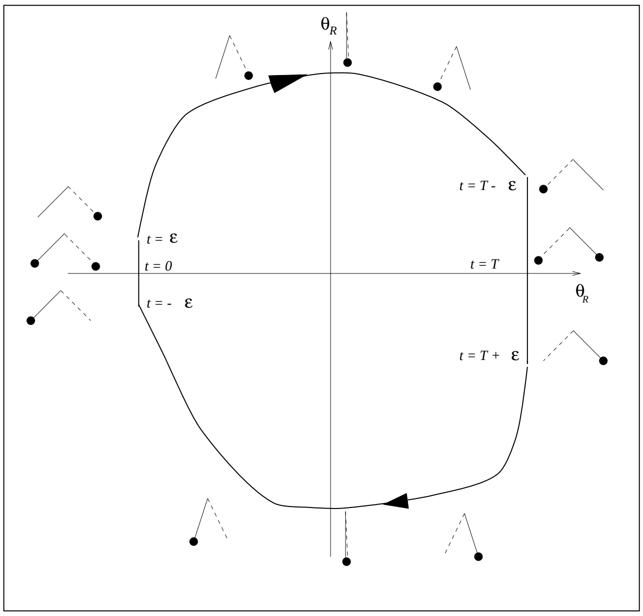


Figure 2: Phase Portrait of a Periodic Walk. This figure corresponds to only one ($\theta_R$, say) of the joint variables of the biped, the actual phase of the system being higher dimensional. One cycle in the figure corresponds to two steps of the robot. In the figure we have indicated some of the time stamps important in the dynamic evolution of the biped. On the outside of the cyclic portrait, the configuration of the biped has been shown with small stick diagrams. In these diagrams, left leg is dotted, right leg is solid, and a black dot at the foot indicates the support leg.

have decided to fix at a known value, it remains to find $T$ such that $det(\boldsymbol{Q} - \boldsymbol{I}_4) = 0$. With this condition and with Equations (4) and (19), we find $\boldsymbol{x}(\varepsilon)$ and then by Equation (3) we determine the slope angle $\phi$. Simulation results [10] confirm the validity of this approach.

The 15 parameters that characterize the motion of this autonomous biped are $\boldsymbol{x}(\varepsilon)$, $\boldsymbol{x}(T-\varepsilon)$, $\boldsymbol{x}(T+\varepsilon)$, $T$, $\phi$, and $\alpha(T)$. Since $\boldsymbol{\theta}(T-\varepsilon) = \boldsymbol{\theta}(T+\varepsilon)$ only 13 of these parameters are independent. These parameters are inter-related by the 8 equations (2), (3), (4), and (20). Note that $\phi$ is a parameter to be found since it determines the motion. We therefore face an under-constrained system where all the motion parameters are to be determined.

#### 4.2.1 Stability of the gait

It is important to distinguish between the stability of the continuous system *during* a step, which is governed by a set of differential equations from that of the gait stability. The first one, not necessarily crucial in this case, is given by classical definitions for non-linear (Equation (1) ) or linear (Equation (9)) systems. The second stability characteristic has to be investigated carefully.

As in [5], we may say that a gait is stable if, *starting from a stationary closed phase portrait of the previous shape, any finite disturbance leads to a limit cycle of similar shape.* Furthermore, if inspite of the disturbance, the systems returns to the original cycle, the gait may be called asymptotically stable.

We have presented the nature of a stable limit cycle (in Figure 3) in the phase plane of one joint variable of the biped. As shown in the figure, the effect of a stable limit cycle in the phase plane will be to attract and absorb the nearby phase trajectories. System starting from a state on the limit cycle will continue to travel on it. The shaded area in the figure indicates the region in which this attracting feature is valid. This shaded area is sometimes called the domain of the limit cycle. It is interesting to note here that a Van der Pol oscillator, for a certain selection of its parameters, the whole phase plane is its domain.

Let us emphasize that this does not require that any of the two curved parts of the cycle be itself a part of a closed limit cycle. Let us finally mention that an interesting mathematical definition and analysis of the stability of systems with impacts may be found in [2].

Here we provide a sketch of the purely passive walk as was studied in [7]. We start from the recurrent equations:

- Dynamics of the $k$th step:

$$\boldsymbol{x}^k(T_k - \varepsilon) = e^{\mathrm{A}_k^T} \boldsymbol{x}^k(\varepsilon) \qquad (23)$$

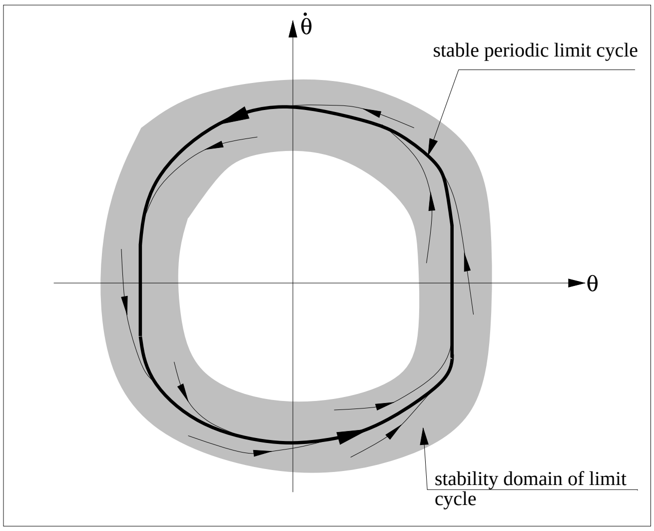


Figure 3: Stable Periodic Walk. As shown in the figure, the effect of a stable limit cycle in the phase plane will be to attract and absorb the nearby phase trajectories. If the system starts from a certain state on the limit cycle, it will continue to travel on it. The shaded area shows the stable domain of the limit cycle.

- Transition condition:

$$\theta_L{}^k(T_k) + \theta_R{}^k(T_k) \quad = \quad -2\phi. \qquad (24)$$

- New step

$$\boldsymbol{\theta}^{k+1}(T_k + \varepsilon) = \boldsymbol{\theta}^k(T_k - \varepsilon) \qquad (25)$$

$$\dot{\boldsymbol{\theta}}^{k+1}(\varepsilon) = \boldsymbol{H}(T_k)\dot{\boldsymbol{\theta}}^k(T_k + \varepsilon) \qquad (26)$$

with

$$\theta_R{}^k(T_k) - \theta_L{}^k(T_k) \quad = \quad 2\alpha(T_k). \quad (27)$$

The stationary gait or the periodic solution, for which the parameters are denoted with superscripted asterices, is given by:

$$\boldsymbol{x}^* = \begin{pmatrix} \boldsymbol{J} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{J} \end{pmatrix} \begin{pmatrix} \boldsymbol{I}_2 & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H} \end{pmatrix} e^{\mathrm{A}T^*} \boldsymbol{x}^* \qquad (28)$$

with

$$\theta_R{}^* + \theta_L{}^* \quad = \quad -2\phi^*. \qquad (29)$$

Since the five equations expressed in (28) and (29) are not sufficient for solving the six unknowns $\boldsymbol{x}^*$, $\phi^*$, $T^*$, we fix the inter-leg angle at the transition: $\theta_R{}^* - \theta_L{}^* = 2\alpha_d$.

For studying stability, we now consider the transient step and write for simplicity:

$$\boldsymbol{x}_k = \boldsymbol{x}^k(0) \qquad (30)$$

By rewriting (23) to (27), we have the following discrete-time evolution equation:

$$\boldsymbol{x}_{k+1} = \boldsymbol{M}(T_k)\boldsymbol{x}_k \qquad (31)$$

with $\theta_{Lk} + \theta_{Rk} = -2\phi = \theta_{L(k+1)} + \theta_{R(k+1)}$ , where $\phi^*$ is given by the stationary solution. We therefore have 6 equations ((31) and (32)) for the 9 unknowns $\boldsymbol{x}_k$, $\boldsymbol{x}_{k+1}$, $T_k$. By setting

$$\delta\alpha_k = |\theta_{Rk} - \theta_{Lk}| \; - \; |\theta_R{}^* - \theta_L{}^*| \qquad (32)$$

$$\delta v_k = \boldsymbol{J}(\dot{\boldsymbol{\theta}}_k - \dot{\boldsymbol{\theta}}^*) \qquad (33)$$

and linearizing all with respect to $T_k$, we finally get 3 transient evolution equations of the form:

$$\delta\alpha_k = |\theta_{Rk} - \theta_{Lk}| \; - \; |\theta_R{}^* - \theta_L{}^*| \qquad (34)$$

$$\begin{pmatrix} \delta\alpha_{k+1} \\ \delta v_{k+1} \end{pmatrix} = \boldsymbol{L} \begin{pmatrix} \delta\alpha_k \\ \delta v_k \end{pmatrix} \qquad (35)$$

which is stable if the eigenvalues of $\boldsymbol{L}$ are inside the unit circle. Let us emphasize that this result comes from strong linearizations and is therefore valid only very locally around the periodic solution. Let us also mention that this condition may also be obtained from the study of Poincare first return map which is one of the topics of our current study.

# 5 Control scheme: use of constant torques with slope estimation

The idea of this simple control approach is the following. First, it is conjectured that the passive periodic gait furnished by a biped robot (such as demonstrated by McGeer) walking down a ramp is optimal in some sense. A natural idea is therefore to try to reproduce this behavior as close as possible using an active control scheme by employing joint torques.

Let us first recall that, in finding the autonomous steady state (see Section 4), we had an undetermination of order 1 when considering the slope angle as unknown. This was solved by setting the value of the inter-leg angle at the transition, $\alpha$. Consequently, the period $T$ was found, then the slope angle $\phi$ and all the other walk parameters. In the present case, we would like to have the robot walking steadily with a given "velocity". Velocity may be defined as the distance between two successive ground contacts of the swing foot divided by the step duration. Given this ideal value, $v^*$, we obtain immediately the related values of $\alpha^*$, $\phi^*$ and $T^*$ by using the plots which explore the steady-state parameters in various cases, (see Figure 4 for an example) as in [7] or [10].

In both cases, the ideal steady-state parameters of the passive walk are therefore uniquely defined. From Equation (16) we see that the corresponding control is given by:

$$\boldsymbol{u} = \boldsymbol{B}^{-1} \begin{pmatrix} p \\ q \end{pmatrix} g(\phi - \phi^*) \qquad (36)$$

where $\boldsymbol{u}$ is the vector {hip torque; support ankle torque}, $\boldsymbol{B} = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}$ and $\phi$ is the actual slope angle. In fact, we would like the system to keep its velocity despite small variations of the slope. The previous control would therefore become:

$$\boldsymbol{u}(t) = \boldsymbol{B}^{-1} \begin{pmatrix} p \\ q \end{pmatrix} g(\hat{\phi}(t) - \phi^*) \qquad (37)$$

where $\hat{\phi}(t)$ is some *estimation* of the actual slope angle.

The way of conducting such an estimation depends on the kind of available measurements. We may consider three cases.

1. The support ankle angle $\theta_1'$ and the inclination of the support leg with respect to vertical are both measured. The estimated angle $\hat{\phi}(t)$ may therefore be obtained by filtering the difference of the two measurements with reinitializing of the process after every transition.
2. Only the support ankle angle $\theta_1'$ is measured. The angle $\hat{\phi}(t)$ may be reconstructed, using a state model of $\phi(t)$, through an appropriate observer during every step.
3. Only the inclinations of the legs are measured. In that case, it is needed to use the information available at the transition in order to estimate $\phi$. Let us examine this situation more carefully:

If, at step $k$, the estimated slope angle is not the right one, the system will behave like if it was a passive biped on a ramp with angle $\phi^* + \phi - \hat{\phi}(t)$, where $\hat{\phi}(t) = \hat{\phi}_k$, constant on one step. Assuming we stay inside the passive stability region, the obtained velocity is not the right one and, since the solution is unique, the velocity error is an image of the slope angle error. We may therefore update the estimation as:

$$\hat{\phi}_{k+1} = \hat{\phi}_k + \lambda \epsilon_k \qquad (38)$$

where $\lambda$ is a positive scalar gain, and $\epsilon_k = \frac{|\alpha_k|}{T_k} - \frac{|\alpha^*|}{T^*}$ (more simply, it appears that the step period is almost constant in practice, and therefore that the inter-leg angle error $\epsilon_k = |\alpha_k| - |\alpha^*|$, might be sufficient).

Finally, it is easy to see that Equation (38) is equivalent to update the control as:

$$\boldsymbol{u}_{k+1} = \boldsymbol{u}_k + \lambda \boldsymbol{B}^{-1} \begin{pmatrix} p \\ q \end{pmatrix} g \epsilon_k \qquad (39)$$

It is, however, necessary to determine the condition on $\lambda$ which would ensure the stability of the discrete-time system under this control law.

# 6 Conclusions

This paper reported the theoretical study of a control scheme for biped robots. The work is under progress, and it will be supplemented in a major way by simulation. The basic idea of the approach is to take advantage of passive walking in order to derive simple control schemes for active periodic stationary gaits. In fact, this work is the starting point for further studies; effective stability results remain to be obtained, especially in the case where realistic nonlinear aspects are considered. It will also be interesting to see if efficient control laws largely based on hip torque can be obtained, since ankle torques are not always available. Finally, we would also like to extend this approach of global walk control to more realistic robots, with several joints. Our general motive is to avoid, as far as possible, the tracking of explicit joint trajectories.

# 7 Appendix

**Details of the dynamic equations** As expressed in Equation (1) the terms of the dynamic equations during the swing stage of our biped robot are:

$$\mathrm{M}(`) = \begin{bmatrix} mb^2 & -mlb\cos(\theta_r - \theta_l) \\ -mlb\cos(\theta_r - \theta_l) & (m_H + m)l^2 + ma^2 \end{bmatrix}$$

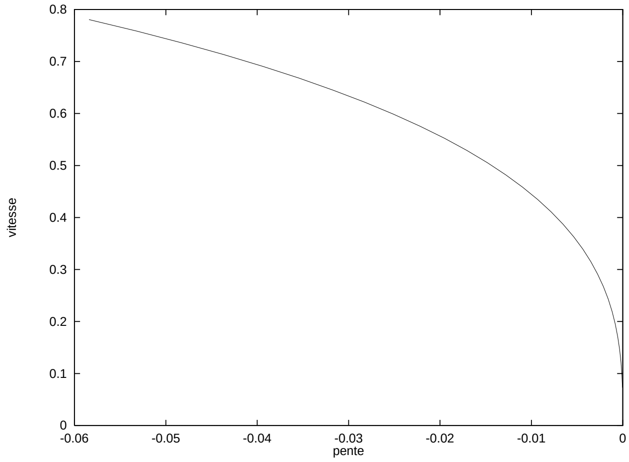


Figure 4: A simulation plot of velocity Vs. slope angle of periodic biped motion, taken from [10]. The curve is monotonic.

$$\mathrm{N}(\grave{}) = \begin{bmatrix} 0 & mlb\sin(\theta_r - \theta_l) \\ -mlb\sin(\theta_r - \theta_l) & 0 \end{bmatrix}$$

$$\mathrm{g}(\grave{}) = \begin{bmatrix} mb\sin\theta_l \\ -(m_H l + ma + ml)\sin\theta_r \end{bmatrix}$$

$$\mathrm{S} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

**Details of the transition equations** The transition equations is of the form $\boldsymbol{Q}(\alpha)\dot{\boldsymbol{\theta}}(T+\varepsilon) = \boldsymbol{P}(\alpha)\dot{\boldsymbol{\theta}}(T-\varepsilon)$. Comparing this equation with the Equation 2 we can write that $\boldsymbol{H}(\alpha) = \boldsymbol{Q}(\alpha)^{-1}\boldsymbol{P}(\alpha)$. The matrices $\boldsymbol{P}(\alpha)$ and $\boldsymbol{Q}(\alpha)$ are of the forms:

$$\mathrm{P}(\alpha) = \begin{bmatrix} (m_H l^2 + 2ml^2)\cos 2\alpha \\ -mab - 2mbl\cos 2\alpha & -mab \\ -mab & 0 \end{bmatrix}$$

$$\mathrm{Q}(\alpha) = \begin{bmatrix} mb^2 - mbl\cos 2\alpha & (ml^2 + ma^2 + m_H l^2) \\ & -mbl\cos 2\alpha \\ -mb^2 & -mbl\cos 2\alpha \end{bmatrix}$$

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