COMPARATIVE STUDY OF METHODS FOR ENERGY-OPTIMAL GAIT GENERATION FOR BIPED ROBOTS

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Abstract In this paper we compare three methods for the energy optimal gait generation for biped robots during the single support phase. The first approach searches for unconstrained trajectories generated by piecewise constant inputs, the second approach constrains the Cartesian trajectories of the swing foot and the hip of the robot to a class of time-polynomial functions, and the last method approximates the robot joint trajectories by a truncated Fourier frequency series. Using a simplified robot dynamics that ignore the centripetal and Coriolis terms, these methods are compared according to the input energy and the initial mechanical energy. The numerical study presented here shows that for an equivalent amount of computational burden, the unconstrained method provides motions with the lowest input energy. Furthermore, it also provides the initial velocities that generate ballistic motions with almost zero input energy.

Keywords Biped robot, energy optimal gait, piecewise-constant control.

Introduction

Recently, many studies have been devoted to locomotion, path planning and control of biped robots [1]-[8]. The main motivations for using walking robots rather than the more conventional wheeled robots for certain tasks are their locomotion capability on irregular surfaces and their versatility in negotiating larger obstacles.

Since mobile robots need to carry their own energy source a lower rate of energy consumption would directly contribute to a longer work cycle. In particular, the current generation legged robots require more control energy than the comparable wheeled robots and this is an important issue to be resolved before the use of legged robots is practically viable. So, the interest of characterizing low energy trajectories appears natural [1] [2] [3] [11]. It is not too unreasonable to expect that their are locomotion gaits for biped robots that consume very little energy. This is reinforced by the biomechanical analysis of natural human gait [10] and practical [9] and numerical [7] results of simple passive biped robot models that can walk down unaided on an inclined slope.

The problem of finding these low energy gaits for a more complex robot model is not trivial and only a few partial analytical results are available up to now, see [6]. Only few papers deal with the energy optimal gait generation while time optimal problem is more commonly treated. Previous works searching for numerical solutions were based on time-polynomial approximations [2] [3], or Fourier expansions [1], or on a combination of both [11]. A nice and quite complete treatment of the application of the optimal programming to human locomotion is given in [5], where penalty functions are used to minimize the total mechanical work done. This technique is now superseded by more recent numerical optimization algorithms. It can be noticed that the concatenation of several phases, with impulsive loads, makes the optimal control problem even harder.

In [4], look for ballistic trajectories through impulsive control which brings the system to required initial conditions to perform the natural gait. This impulsive control is supposed to be instantaneous and it is applied during the impact phase.

In this paper we propose an alternative method to generate ballistic motion of the biped by supposing that a control exists all through during the motion (as opposed to the methods of [4] [8]). The optimization process generates the initial and terminal velocities that correspond to the minimum-energy motions. This approach is compared to the two previously proposed methods based on an optimization over a restricted class of joint trajectory functions.

Problem formulation

The complete dynamics of a biped robot faithfully mimicking the human gait will be rather involved. The human gait cycle is divided into two phases: the single support phase or the swing phase (one foot on the ground and the other foot swinging) and the double support phase (both feet on the ground). The transition from the single support to the double support phase, also called the *contact phase*, is associated with the heel of the front foot impacting with the ground. The transition from the double support to the single support phase of the next step, also called the *take-off phase*, is caused when the toe of the rear foot leaves the ground. The dynamic equations of a robot consisting of all the described phases is composed of ordinary differential equations for swing stage and algebraic equations for the transition stages (the latter are usually modeled as instantaneous phenomena). Moreover, the topology of the kinematic chain making up the robot changes from the single support to the double support phase complicating further the differential equations.

It is not an easy task to choose a kinematic model for the biped that captures the essence of the anthropomorphic gait while keeping the model reasonably simple to allow intuitive insights about its behavior. Admitting the fact that the simplifications may sacrifice some of the subtleties of human motion, we have converged upon a planar four degrees-of-freedom (DOF) biped mechanism as shown in Figure 1. We have assumed that in this model the trunk will be upright during the walk. This seems reasonable because the trunk's maximal excursion from the vertical axis is about 20mm at the pelvis point, as reported in [11]. We will consider that the center of mass of the trunk is located at the hip, that is at the center of rotation of the third joint.

The foot of the swing leg is considered massless thereby obviating motor in the swing leg ankle. This guarantees that our robot model always has 4 actuated joints. In spite of this commonly used assumption (see [3] and [11]) which substantially simplifies the model, foot articulations (tarsus and metatarsus) play an important role during the transition phase in generating and absorbing a significant amount of energy.

The n-DOF model biped robot is obtained by the Euler-Lagrange equations:

$$\boldsymbol{H}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{1}$$

where the vector $\boldsymbol{q} \in R^n$ describes the generalized joint coordinates, $\boldsymbol{H}(\boldsymbol{q})$ is the inertia matrix, $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ is the matrix of centripetal acceleration and Coriolis terms, $\boldsymbol{g}(\boldsymbol{q})$ is the gravity vector, and $\boldsymbol{u}$ is the input torque vector.

From the energy point of view, the forces due to variation of $\boldsymbol{H}(\boldsymbol{q})$ are workless and hence they contribute very little to changes in the energy levels since they have no impact

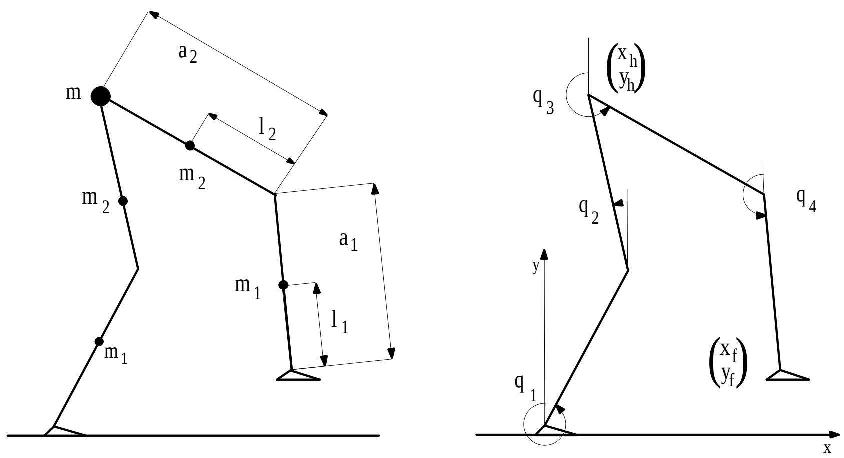

Figure 1: Simplified structure of the 4-DOF biped robot used for our study.

on the time variation of the system Hamiltonian (i.e. $\dot{\boldsymbol{q}}^T \left\{ \frac{1}{2}\dot{\boldsymbol{H}}(\boldsymbol{q}) - \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) \right\} \dot{\boldsymbol{q}} = 0$). They will then not be taken into account in our study. This assumption is reinforced by the fact that gear ratios of the D-C actuators are large enough so that coupling and position dependent terms of the inertia matrix can be ignored. The main nonlinearities considered by our model are the gravity term. The simplified biped dynamics is:

$$\boldsymbol{H}\ddot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{u} \tag{2}$$

where $\boldsymbol{H}$ is a diagonal constant matrix.

The robot dynamics (2), can also be expressed in a state-space description:

$$\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u}) \quad t \in [0, T] \tag{3}$$

where $\boldsymbol{x} = [\boldsymbol{x}_1^T, \boldsymbol{x}_2^T]^T$, the state vector, is composed of the position vector, $\boldsymbol{x}_1$, and the velocity vector $\boldsymbol{x}_2$. Biped motion should be defined within an admissible set of joint positions Ω_x: $\boldsymbol{x}_1 \in \Omega_x$ that describes a region in the joint configuration space, where some physical restrictions are imposed (i.e. legs should not cross the ground level, avoid singular configuration, etc.). The specified boundary conditions for the optimization are the initial and the terminal joint angles, $\boldsymbol{x}_1(0)$ and $\boldsymbol{x}_1(T)$, respectively, and the time interval of the swing phase T. The initial and final velocities $\boldsymbol{x}_2(0)$ and $\boldsymbol{x}_2(T)$ are free however. They represent an additional degree of freedom giving the possibility for the optimization procedure to generate low-energy trajectories (with zero cost).

For optimization we will use the following cost criterion:

$$J = \int_0^T \boldsymbol{u}^T \boldsymbol{u} \, dt \tag{4}$$

the minimization of which can be shown to be equivalent to the minimization of the injected energy to the robot, other losses are neglected.

We should also define the admissible set $\mathcal{U}$ of feasible control $\boldsymbol{u}(t)$ which defines a class of bounded signals with finite energy in the time interval $[0, T]$: $\boldsymbol{u} \in \mathcal{U} \triangleq \mathcal{L}_2^e \cap \mathcal{L}_\infty^e$ where $\mathcal{L}_2^e$, and $\mathcal{L}_\infty^e$ stand for the extended $\mathcal{L}_2$, and $\mathcal{L}_\infty$ spaces, respectively.

Problem 1 *Given the initial and final joint angles $\boldsymbol{x}_1(0) = \boldsymbol{x}_{10} \in \Omega_x$, $\boldsymbol{x}_1(T) = \boldsymbol{x}_{1T} \in \Omega_x$ and the time interval T, the problem is to find the optimal sequence $\boldsymbol{u}^\star(t) \in \mathcal{U}$, minimizing the cost function J (eq 4), such that it steers the system (3) from $\boldsymbol{x}_{10}$ to $\boldsymbol{x}_{1T}$ complying with the restriction $\boldsymbol{x}_1(t) \in \Omega_x$. Or equivalently:*

$$\begin{cases} \min\limits_{\boldsymbol{u}\in\mathcal{U}} \; J(\boldsymbol{u}) = \int_0^T \boldsymbol{u}^T\boldsymbol{u}\, dt \\ \\ under \begin{cases} \dot{\boldsymbol{x}}(t) = f(\boldsymbol{x}(t), \boldsymbol{u}(t)) \\ \boldsymbol{x}_1(t) \in \Omega_x \end{cases} \\ \\ given \; (\boldsymbol{x}_{10}, \boldsymbol{x}_{1T}, T) \end{cases} \tag{5}$$

Optimization methods

In this section, two classes of optimization methods will be discussed. In the first, the control input $\boldsymbol{u}(t)$ is assumed to be a series of constant values during the time-intervals $\Delta t = T/N$. We refer to this method as the unconstrained piecewise constant control, since the sequence of $\boldsymbol{q}(t)$ produced by this control input is not explicitly restricted to belong to any particular time or frequency function. In the second class of optimization methods, called the constrained methods, the Cartesian vector $\boldsymbol{y}(t)$ or, equivalently, the joint position vector $\boldsymbol{q}(t)$ is constrained to belong to a time-series polynomial expansion or to a Fourier frequency approximation. The control input in both the classes obey the inverse dynamic equations.

Unconstrained and piecewise constant method. The dynamic optimization problem 1, can be transformed into a static optimization problem as follows. First, assume that the control sequence $\boldsymbol{u}(t)$ is piecewise constant. Let N be the number of time-intervals and $\Delta t = T/N$ the time-length of these intervals. The only restriction on the sequence $\{\boldsymbol{u}(k)\}_{k=0}^{N-1}$ is that each element belongs to $\mathcal{U}$.

Let $\boldsymbol{U} \in R^{n\times N}$ be the input matrix gathering the input vector sequence $\boldsymbol{u}(k)$, i.e.

$$\boldsymbol{U} = [\boldsymbol{u}_0, \boldsymbol{u}_1, \boldsymbol{u}_2, \ldots, \boldsymbol{u}_{N-1}] \tag{6}$$

then the cost function (4) can be rewritten as:

$$\mathcal{C} = \sum_{k=0}^{N-1} \boldsymbol{u}(k)^T\boldsymbol{u}(k)\; \Delta t \tag{7}$$

Then, by approximating $\dot{\boldsymbol{x}}$ as,

$$\dot{\boldsymbol{x}} = \frac{\boldsymbol{x}(k+1) - \boldsymbol{x}(k)}{\Delta t}$$

in the state equation (3), we can obtain the following implicit discrete-time nonlinear representation

$$\boldsymbol{x}(k+1) = \boldsymbol{x}(k) + \Delta t f(\boldsymbol{u}(k), \boldsymbol{x}(k))$$

By induction, it is possible to express the state $\boldsymbol{x}$ at the instant N as a function of the initial state $\boldsymbol{x}(0)$ and the series $\boldsymbol{u}(0), \boldsymbol{u}(1), \cdots \boldsymbol{u}(N-1)$, i.e.

$$\boldsymbol{x}(N) = F(\boldsymbol{x}(0), \boldsymbol{u}(0), \cdots, \boldsymbol{u}(N-1)) = F(\boldsymbol{x}(0), \boldsymbol{U}) \tag{8}$$

where the operator F is defined as:

$$F = f \circ f \cdots \circ f(\boldsymbol{x}(0), \boldsymbol{u}(0)) \tag{9}$$

The dynamic optimization problem can now be transformed into the following static optimization one.

Problem 2 *Given the initial and final joint angles $\boldsymbol{x}_1(0) \in \Omega_x$, $\boldsymbol{x}_1(N) \in \Omega_x$ and the time interval T, the problem is to find the optimal value for $\boldsymbol{u}^\star(t) \in \mathcal{U}$, minimizing the cost function $\mathcal{C}$ (7), such that it steers the system (3) from $\boldsymbol{x}_1(0)$ to $\boldsymbol{x}_1(N)$ complying with the restriction $\boldsymbol{x}_1 \in \Omega_x$. Or equivalently:*

$$\begin{cases} \min\limits_{\boldsymbol{U} \in \mathcal{U}} \ \mathcal{C}(\boldsymbol{u}) = \sum\limits_{k=0}^{N-1} \boldsymbol{u}(k)^T \boldsymbol{u}(k)\ \Delta t \\ under \begin{cases} \boldsymbol{x}(N) = F(\boldsymbol{x}(0), \boldsymbol{U}) \\ \boldsymbol{x}_1 \in \Omega_x \end{cases} \\ given\ (\boldsymbol{x}_1(0), \boldsymbol{x}_1(N), T, N) \end{cases} \tag{10}$$

Note that, as above, the initial and final velocity are free in our problem, and should thus be generated as a result of optimization. Note also that the discretization has been performed on the $2n$ dimensional state vector $\boldsymbol{x}$ and not on the equation (2) to reduce discretization errors due to approximation of the second-order time derivative.

Polynomial Approximation. Let $\boldsymbol{y}(t)$ be defined as:

$$\boldsymbol{y}(t) = \begin{pmatrix} z_h \\ x_h \\ z_f \\ x_f \end{pmatrix}$$

where (z_h, x_h), and (z_f, x_f), are the Cartesian coordinates of the hip and foot, respectively (see Fig. 1), then robot motion can be specified in terms of these coordinates. They are related to the joint angles by the mapping $\boldsymbol{y} = W(\boldsymbol{q})$. As proposed in [3] and [2], the optimal trajectory in $\boldsymbol{y}(t)$ can be assumed to be approximated by an m-order time-polynomial of the form:

$$\boldsymbol{y}(t) = \boldsymbol{p}_0 + \boldsymbol{p}_1 t + \cdots + \boldsymbol{p}_m t^m.$$

With initial and final position given, $\boldsymbol{p}_0$ is uniquely defined. The remaining parameters $\boldsymbol{P} = [\boldsymbol{p}_1 \cdots \boldsymbol{p}_m]$ should be determined by the optimization procedure as explained below.

From the inverse kinematics we have:

$$\boldsymbol{q}(t) = W^{-1}(\boldsymbol{y})\ ;\ \dot{\boldsymbol{q}} = \boldsymbol{J}^{-1}\dot{\boldsymbol{y}}\ ;\ \ddot{\boldsymbol{q}} = \boldsymbol{J}^{-1}[\ddot{\boldsymbol{y}} - \dot{\boldsymbol{J}}\boldsymbol{J}^{-1}\dot{\boldsymbol{y}}] \tag{11}$$

where $\boldsymbol{J}$ is the Jacobian matrix which is full rank (in this case where $\boldsymbol{J}$ is quadratic, it will be non-singular) for all $\boldsymbol{q} \in \Omega_x$. Note that singular configurations are never reached during the human walk. Combining (2) and (11), and using the time approximation given above, $\boldsymbol{u}(t)$ can be written as a function of the unknown parameters $\boldsymbol{P}$:

$$\boldsymbol{u} = \boldsymbol{H}\boldsymbol{J}^{-1}\ddot{\boldsymbol{y}} - \boldsymbol{H}\boldsymbol{J}^{-1}\dot{\boldsymbol{J}}\boldsymbol{J}^{-1}\dot{\boldsymbol{y}} + \boldsymbol{g}(W^{-1}(\mathrm{y})) \tag{12}$$

$$= \Phi_p(\boldsymbol{P}, t) \tag{13}$$

Problem 1, can then be reformulated as:

Problem 3 *Given the initial and final joint angles $\boldsymbol{x}_1(0) = \boldsymbol{x}_{10} \in \Omega_x$, $\boldsymbol{x}_1(T) = \boldsymbol{x}_{1T} \in \Omega_x$ and the time interval T, the problem is to find the optimal parameters $\boldsymbol{P}^\star$, minimizing* *the cost function J (eq 4), such that it steers the system (3) from* $\boldsymbol{x}_{10}$ *to* $\boldsymbol{x}_{1T}$ *complying with the restriction* $\boldsymbol{x}_1(t) \in \Omega_x$. *Or equivalently:*

$$\begin{cases} \min\limits_{\boldsymbol{P}} \sum\limits_{k=0}^{N-1} \Phi_p(\boldsymbol{P},k)^T \Phi_p(\boldsymbol{P},k)\ \Delta t \\ \\ under\ \boldsymbol{x}_1 \in \Omega_x \\ \\ given\ (\boldsymbol{x}_1(0), \boldsymbol{x}_1(N), T, N) \end{cases} \tag{14}$$

where $\mathcal{U}_p$ is the set of functions included in $\mathcal{U}$ generated by equation (13) for all $\boldsymbol{P} \in R^{n \times m}$. Note also that the initial and final conditions in the joint coordinates are obtained (uniquely) from the initial conditions on the Cartesian coordinates.

Fourier Approximation. Instead of constraining the Cartesian trajectories of the robot one can also constrain its joint trajectories and one straigtforward way to do this is with a truncated Fourier series. This approximation may be called the frequency approximation, and it can be expressed as:

$$\boldsymbol{q}(t) = \boldsymbol{a}_0 + \sum_{k=1}^{K} \boldsymbol{a}_k cos(k\omega t) + \sum_{k=1}^{K} \boldsymbol{b}_k sin(k\omega t)$$

where the $\boldsymbol{a}_i$ and the $\boldsymbol{b}_i$ are the Fourier coefficients and ω is the step base frequency. If only the single support phase is considered then $\omega = 2\pi/T$. However, in this case the trajectory has an important bias between initial and final joint positions. This type of trajectories is thus poorly approximated by only using the expansion given above. To solve this problem it has been suggested in [11] to add a time-polynomial to absorb differences between the initial and final conditions without over-increasing the order of the Fourier approximation. For instance, adding a first-order polynomial gives,

$$\begin{aligned} \boldsymbol{q}(t) &= \boldsymbol{a}_0 + \boldsymbol{c}t + \sum_{k=1}^{K} (\boldsymbol{a}_k cos(k\omega t) + \boldsymbol{b}_k sin(k\omega t)) \\ &= \phi_f(\boldsymbol{D}, t) \end{aligned} \tag{15}$$

$\boldsymbol{D} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \ldots, \boldsymbol{a}_K, \boldsymbol{b}_1, \boldsymbol{b}_2, \ldots, \boldsymbol{b}_K, \boldsymbol{c}]$ is the unknown parameter matrix to be determined.

The vectors $\dot{\boldsymbol{q}}$ and $\ddot{\boldsymbol{q}}$ are computed from (15) and substituted into (2). They enable the control $\boldsymbol{u}$ to be written as:

$$\boldsymbol{u} = \Phi_f(\boldsymbol{D}, t) \tag{16}$$

This equation defines, for all matrices $\boldsymbol{D}$ with bounded entries, a class of signals $\mathcal{U}_f \subset \mathcal{U}$, in which the search for optimal $\boldsymbol{u}^\star$ will be performed.

Our problem is now formulated as follows:

Problem 4 *Given the initial and final joint angles* $\boldsymbol{x}_1(0) = \boldsymbol{x}_{10} \in \Omega_x$, $\boldsymbol{x}_1(T) = \boldsymbol{x}_{1T} \in \Omega_x$ *and the time interval T, the problem is to find the optimal parameters* $\boldsymbol{D}^\star$, *minimizing the cost function J (eq 4), such that it steers the system (3) from* $\boldsymbol{x}_{10}$ *to* $\boldsymbol{x}_{1T}$ *complying with the restriction* $\boldsymbol{x}_1(t) \in \Omega_x$. *Or equivalently:*

$$\begin{cases} \min\limits_{\boldsymbol{D}} \sum\limits_{k=0}^{N-1} \Phi_f(\boldsymbol{D},k)^T \Phi_f(\boldsymbol{D},k)\ \Delta t \\ \\ under\ \ \boldsymbol{x}_1(k) \in \Omega_x \\ \\ given\ (\boldsymbol{x}_1(0), \boldsymbol{x}_1(N), T, N) \end{cases} \tag{17}$$

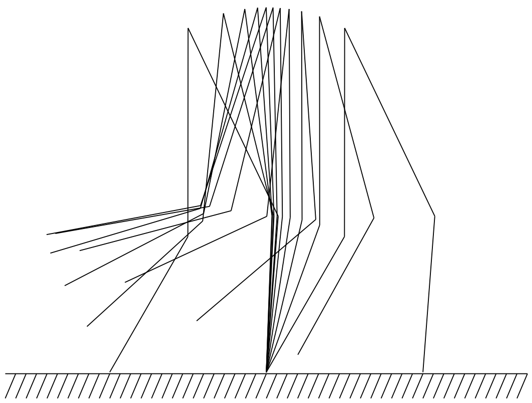

Figure 2: Walk with piecewise constant control.

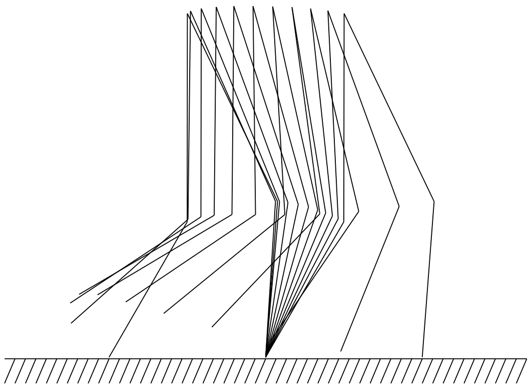

Figure 3: Walk with polynomial approximation.

Results.
The optimization method used here is a Sequential Method Programming based on Kuhn-Tucker equations. This method solves at each iteration a Quadratic Programming subproblem (QP) which involves a quadratic approximation of the Lagrangian function composed by the cost function and the constraints which are premultiplied by the Lagrange multipliers.

In order to compare the different optimization methods, the following comparison criteria are used:

- Input Energy. The cost function J (eq 4) represents the total control energy needed to perform the single support phase. This measure quantifies the energy consumption of the biped over one step without considering the energy consumed during the transition from one step to the next.

- Initial Energy. If the injected energy is zero during the time step interval (i.e., the motion is passive in the sense that it is generated by a zero control input), it is important to take note of the initial value of the energy. This quantity represents the energy level necessary to bring the system to the initial state required to perform the ballistic gait. The total energy needed during a step is the sum of the energy during the single-support phase and the energy during transition.

- Computational Burden. The CPU computation time and the number of parameters to be optimized are also considered as a function of the total amount of computation burden associated to each optimization method.

For the simulation results presented in this section, we have used as the walk constants the step length $S = 0.6m$ and the step period $T = 1s$. Initial and final positions are chosen to be symmetrical.

In figures (2) (3) and (4) we can observe the optimal gait obtained respectively with piecewise constant time control(PCTC), polynomial approximation and Fourier expansion respectively. Some differences can be observed; the foot trajectories are similar in Fourier expansion and the polynomial approximation, but quite different in the PCTC, whereas the hip trajectories are similar in the Fourier expansion and the PCTC, but different in the polynomial approximation. These trajectories are shown in more detail in figure (5).

The control cost (J) and initial energy ($\mathcal{H}(0)$) associated with each of these methods are shown in Table I. It can be seen that the energy consumption is close to zero with

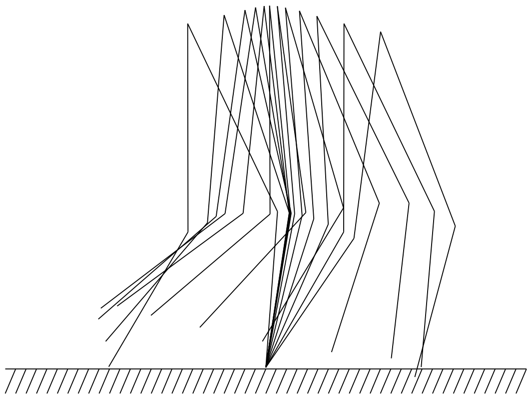

Figure 4: Walk with Fourier series approximation.

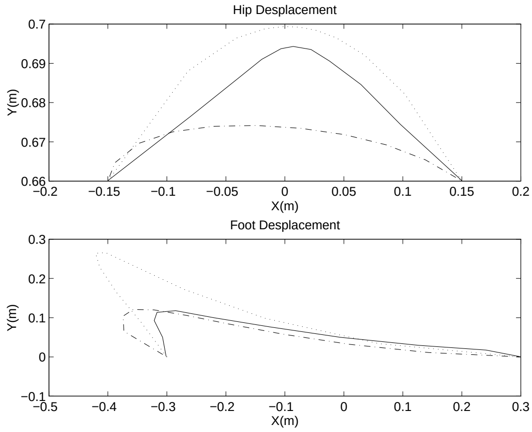

Figure 5: Hip end Foot trajectories −Fourier method − · −·Polynomial method · · ·Piecewise constant method.

METHOD	Cost J $(N^2.m^2.s)$	$\mathcal{H}_0$ $(N^2.m^2.s)$	Par. No.	CPU time s
Piecewise constant	2.10^{-7}	53.2	$n \times N = 40$	37
Polynomial Approximation	2121	47.9	$n \times m = 20$	137
Fourier Expansion	1565	48.1	$n \times (2K+2)$ =40	451

Table I: Comparison of the Optimization Methods Based on the Three Proposed Criteria. the

piecewise constant control. This implies that this formulation finds initial conditions of velocity so that the gait is passive during the single support phase. The energy required to achieve these initial conditions is approximately the same as the one required in the other two methods. However, the initial directions of the velocity joint vectors turn out to be quite different. In PCTC optimal gait, the swing leg behavior is like a pendulum: the initial velocity carries the foot sufficiently high in order to arrive at the final position through a "free" movement. Fig(6) shows the time evolutions of the control inputs.

The number of optimization parameters of these methods are quite equivalently (see Table I, column 4), however CPU-time of the PCTC method is much smaller than in the other two. By looking at this table, the PCTC appears superior to the other two methods in all the comparison criteria.

Conclusions and Further Extensions.

This paper presented and compared three methods for optimal-energy gait generation for biped robots. These methods are: piecewise constant inputs, time-polynomial approximation and Fourier expansion. The comparison was performed on the basis of injected energy, initial energy and computational burden. The numerical study presented here shows that the piecewise constant input method is superior to the other two approaches in terms of energy and CPU-time. Moreover, this method was able to find initial velocities that generate ballistic motions with almost zero injected energy. Hence the method was able to find passive motions. The study was only concerned with the single support phase and the transition phases. We have already developed a model and are currently studying a complete gait cycle including the double support phase. It is interesting to investigate the possibility of finding other basis functions to approximate the walking gaits, and to

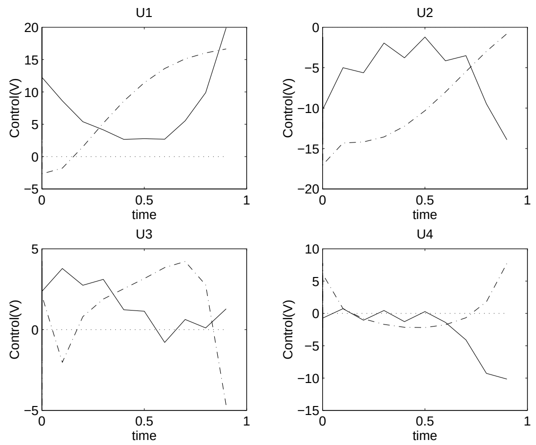

Figure 6: Input control during a step —Fourier method — · —·Polynomial method · · ·Piecewise constant method.

study the potential benefits of introducing passive elements in the joint articulations [1], which appear as a natural way of enhancing a passive walk.

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