MECHANICAL COMPUTATION FOR PASSIVE FORCE CONTROL

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ABSTRACT

Force control implemented by a passive mechanical device (perhaps a wrist) has inherent advantages over active implementations. A passive mechanical device can regain some of the versatility of its active counterpart if it incorporates mechanical elements with programmable parameters, e.g. damping coefficients or spring stiffnesses. We wish to characterize the range of accommodation matrices that a passive device may be programmed to possess.

Here we review the known theoretical limits on the accommodation (inverse damping) matrices that any linear system of programmable dampers may adopt. Recent results [22, 26] show that such matrices are well suited to force-guided assembly. However, even with fully adjustable damping constants a mechanical device of fixed geometric design can attain only a subset of the all accommodation matrices.

In this work we describe the set of attainable accommodation matrices, and show that each such matrix can be composed of a positive linear combinations of a fixed set of basis matrices. We show how the damping coefficients can be chosen to achieve a desired accommodation matrix, i.e. how to program this mechanical computer. We compare the space of attainable matrices to the space of all matrices, and suggest a method of visualizing it in low-dimensional examples.

1.0 MOTIVATION

In precision tasks such as robotic assembly, force control seems to be the natural choice and is widely believed to be superior to pure position control. In a typical force control scheme the motion of a robot is guided (according to a pre-defined control law) by the forces the robot encounters while interacting with the environment. The performance of such a scheme depends on the particular force control law and the nature of its implementation.

Force control laws may be broadly classified into two types – passive laws and active laws. A passive law describes a force-motion behavior that may, in principle, be exhibited by some passive physical system. Active laws, on the other hand, require the presence of a power source in the system.

The robotics community is recently taking a fresh look at the property of *passivity* in a controlled system and the advantages associated with it, especially in energetic interaction with the environment. Passivity of a system guarantees its stability, a result that has been known for a long time [8]. Colgate showed that *only* a passive system remains stable at all frequencies when coupled to an arbitrary passive environment [6].

A passive force control law may be implemented either by a software algorithm or by an unpowered mechanical system. In a software-controlled system, active components (motors) are controlled in such a way that the overall system emulates passive behavior [2, 21, 28]. Unfortunately, the speed and performance of such a system is limited by the control system bandwidth [31], response time of the actuators, and non-collocation of the sensors and the actuators. This motivates the use of passive mechanical devices which, by virtue of their inherent mechanical properties, allow the implementation of passive force control laws.

Unpowered devices with fixed mechanical properties lack the versatility offered by software controllers. An attractive alternative for implementing force control laws is the use of mechanical devices with user-programmable properties. Such a device is able to regain some of the versatility of its software counterpart.

Rather than involving the whole robot arm for the fine positionings necessary for the completion of most assembly tasks, using a low inertia robotic wrist mounted at the end of the robot arm will have the advantage of higher mechanical bandwidth. A robotic wrist made up of passive physical components such as springs and dampers enjoys the additional advantage of simplicity in design and guaranteed stability. An example of such a wrist is the remote center of compliance (RCC) wrist which is successful in peg-in-hole assembly.

We have an ongoing interest in the idea of "programming" a manipulator's linear damping characteristics so that for some classes of assembly tasks the forces which arise from positional errors of the mating parts in assembly naturally result in the motions which correct the errors. These types of tasks can be performed under force control alone, with no other sensory information [22, 25] .

In the next section we describe the scope and summarize the contribution of this paper. In Section 3.0 we review related research. Section 4.0 discusses the known results derived from electrical network theory on the set of *realizable* accommodation matrices; those matrices that can be attained by proper selection of dampers. We then identify the set of *synthesizable* accommodation matrices, a subset of realizable matrices, which can be programmed into a passive device in a routine way. Section 5.0 details various ways of geometrically visualizing the class of synthesizable matrices. In Section 6.0 we transport the joint-space results to task-space in order to derive synthesizability conditions in the later. Several examples are given in this section.

2.0 SCOPE AND CONTRIBUTION

Imagine a passive hydraulic mechanism employed as a robot wrist. Suppose a workpiece is held by the wrist and is moving with a nominal velocity $\boldsymbol{v_0}$ in the absence of any assembly force. $\boldsymbol{v_0}$ is therefore the velocity of the robot/workpiece under pure position control. When the workpiece comes in contact with its mating part, its resultant velocity $\boldsymbol{v}$ may be expressed as

$$\boldsymbol{v} = \boldsymbol{v_o} + \boldsymbol{A}\ \boldsymbol{f} \ , \qquad (1)$$

where $\boldsymbol{f}$ is the force resulting from unavoidable positional errors between the mating parts. $\boldsymbol{A}$ is the *accommodation* (inverse damping) matrix of the wrist. It maps forces imparted on the wrist to output velocities. Each of $\boldsymbol{v}$, $\boldsymbol{v_0}$, and $\boldsymbol{f}$ is a 6-vector (translational and rotational velocities, or forces and torques), and $\boldsymbol{A}$ is a 6×6 matrix.

Equation 1 represents the force control law that we intend to implement with programmable passive devices. The control law is essentially an additive modification of the nominal velocity $\boldsymbol{v_0}$ of the robot wrist. The deviation of the wrist motion from the nominal velocity (given by $\boldsymbol{A}\ \boldsymbol{f}$) is a function of the accommodation matrix of the wrist. Our goal is to program the accommodation matrix in such a way that the resultant velocity $\boldsymbol{v}$ reduces relative positional errors between the mating parts. By programming the parameters of the wrist we can potentially implement a range of control laws (represented by a range of $\boldsymbol{A}$).

The force-velocity model adopted in Equation 1 is also known as the *generalized damper model* of a system. For the hydraulic devices that we consider here, damping/accommodation properties dominate over inertia and stiffness properties. Use of the generalized damper model is therefore justified.

Examples:

The passive devices considered as examples in this paper consist of a set of unpowered hydraulic cylinders with their ports interconnected via a hydraulic network of programmable-damping constrictions. Fig. 1 shows a sketch of a 2 DOF passive mechanism. It consists of two hydraulic cylinders in a parallel configuration. The constrictions of the interconnecting hydraulic network let one "program" a desired accommodation matrix of the manipulator[1]. The central question this paper addresses is: assuming that the passive dampers in the network one can be continuously changed, what range of accommodation matrices can this device assume ?

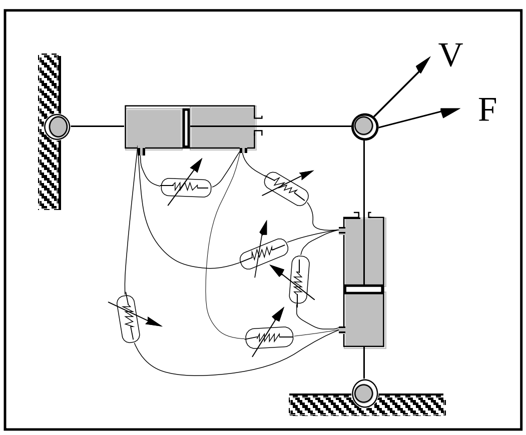

Fig. 1. A simple parallel 2 DOF passive mechanism. The ports of the hydraulic cylinders are interconnected through constrictions with tunable damping.

Fig. 2 shows a Stewart platform based robotic wrist consisting of six parallel hydraulic cylinders. The overall accommodation matrix of the wrist (task-space accommodation matrix) is related to the hydraulic conductance matrix of the network (joint-space accommodation matrix) as well as the spatial layout of the cylinders. Assuming that the wrist executes small motions about a nominal position, the change in its spatial geometry is considered relatively insignificant. The overall accommodation matrix of the wrist is changed by changing the network parameters.

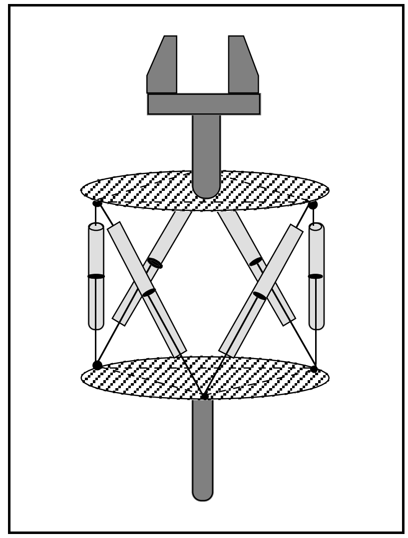

Fig. 2. A six DOF Stewart platform type robotic wrist.

We showed in [10] that such a wrist may be programmed to possess a *center of accommodation* (similar to a *center of compliance*) anywhere in a substantial volume of space around it. The accommodation matrices of interest to us are by no means restricted to those which possess a center of accommodation in the task-space. In fact our recent work on the synthesis of error-corrective accommodation matrices [25] often results in matrices which do not have a center of accommodation (i.e., which may not be diagonalized with respect to any coordinate frame). We wish to characterize the complete range of passive programmable matrices, both diagonal and non-diagonal.

Accommodation matrices which are, *in principle*, attainable with a network of passive dampers are called *realizable* matrices according to network theory [29]. However, realizable accommodation matrices exist for which no routine way of computing the necessary network parameters is available. By *synthesizable* matrices, we refer to those matrices that the wrist can be systematically (algorithmically) programmed to possess. In this paper we identify the class of task-space accommodation matrices which are synthesizable with a fixed geometry hydraulic wrist coupled to a network of programmable passive dampers.

We import an algorithmic programming scheme from the electrical network theory to the mechanical domain and describe the synthesizable conductance matrices of the hydraulic network (joint-space accommodation matrices), called *dominant* matrices. These matrices are projected into task-space through a congruence transformation. All synthesizable task-space matrices are shown to be composed of positive linear combinations of a fixed set of basis matrices.

We describe the topological properties of the sets of synthesizable accommodation matrices in joint-space and in task-space and suggest a way of visualizing these sets for low dimensional matrices. Finally, in an attempt to quantify the versatility of programmable passive devices with fixed

[1] Although robotic arms have some inherent structural accommodation properties, the accommodation of the end-point device will be much higher and will dominate the overall accommodation.

geometry, we compare the "volume" of the synthesizable matrices with that of a standard class of matrices (positive semi-definite matrices).

3.0 RELATED RESEARCH

A systematic analysis of compliance properties of a passive device to implement a particular useful force control law is found in the works of [9, 30]. The device, which is called a remote center of compliance (RCC) wrist, however, lacks programmability since it is made up of components of fixed mechanical properties.

Asada and Kakumoto [3] suggested a dynamic RCC wrist for which the *center of inertia* is located in such a way that it helps in controlling the dynamic forces during high-speed assembly. Cutkosky [7] designed a wrist with programmable compliance – a programmable RCC device whose center of compliance may be located anywhere inside a restricted task-space region.

Mason [20] described hybrid control and explicit feedback as useful compliance control schemes for a manipulator. Hogan [14] formulated impedance control which suggests the control of overall dynamic response of a system as opposed to independent position control or force control. A suitable impedance function is chosen in order to achieve a desirable dynamic behavior of the manipulator.

Most of the above work (except [14]) dealt with diagonal compliance or accommodation matrices, matrices having a "center" in the task-space. Schimmels and Peshkin [26] formulated a systematic mean of identifying the bounds of force-guided assembly and a systematic approach to the design of a manipulator's accommodation that guarantees force-guided assembly. Their synthesis often results in non-diagonal matrices.

Loncaric applies the theory of Lie groups to show that any symmetric stiffness matrix may be designed by using systems consisting of passive linear springs only [19]. Griffis and Duffy model the general spatial stiffness of a coupling between two objects as a 6 DOF passive Stewart platform of springs [11].

4.0 RESULTS FROM ELECTRICAL NETWORK THEORY

Analogues exist among passive devices in different physical domains - electrical, mechanical, hydraulic, etc. Bond graph theory exploits these fundamental analogies in order to model physical systems [17]. There are five different types of passive elements which, in the electrical domain, are known as resistors, capacitors, inductors, transformers, and gyrators. Electrical resistors and hydraulic dampers belong to the same class of elements. Similar analogies exist between electrical capacitors and mechanical springs, between inductors and masses, and between ideal electrical transformers and mechanical levers[2]. Understanding these physical analogies makes it possible to apply results obtained in one physical domain to another.

The most general representation of a linear dynamic system may be in terms of its admittance or impedance matrices. Dynamic behavior of *single-element-kind* mechanical systems may be expressed by special forms of admittance matrices. The behavior of a network of linear springs (often referred to as a generalized spring [19]) is expressed in terms of its compliance matrix. Similarly, the dynamic behaviors of a generalized damper and a generalized mass are described by accommodation matrices and inverse-inertia (or mobility) matrices respectively[3]. One may also generalize the concepts of *center of compliance*, *center of accommodation*, and *center of mobility* to define an overall *center of admittance* [27].

Passive networks satisfy the so-called *passivity condition* which implies that matrices adopted by passive devices must be positive real [29] . If we remove gyrators from a general passive device, the admittance matrix must be symmetric [1] [8]. If we remove capacitors and inductors (or their mechanical analogues) from the possible range of available components, we are left with a positive semidefinite (PSD) matrix, which is a matrix with real elements.

The exclusion of transformers leaves us with a purely resistive circuit which possesses the so called *no-amplification* property. Cederbaum generalized the idea of no-amplification [4] and showed that the accommodation matrix of a purely resistive circuit must be a *paramount matrix*.

Definition:

A real symmetric matrix is said to be <u>paramount</u> if any of its principal minors is not less than the absolute value of any other minor built upon the same rows (or columns) by replacing any number of columns (or rows).

It has been shown that paramountcy is a necessary but unfortunately not a sufficient condition for realizability[4]. There are presumably other restrictions on realizability that have not yet been identified. A sufficient, but overly restrictive, condition for an accommodation matrix to be attainable is that it be *dominant* [29] [18] :

Definition:

A real symmetric matrix is said to be <u>dominant</u> if each of its main diagonal elements is not less than the sum of the absolute values of all the other elements in the same row (or column).

There are in fact examples of networks whose accommodation matrices are not dominant. There are also examples of paramount matrices for which it can be proved that there is no realization. Dominant matrices represent an important class of matrices in synthesis of passive resistive network because there is a methodical procedure of synthesizing *any* dominant matrix. Therefore for our purpose, dominant matrices are classified as the synthesizable matrices. For an actual synthesis procedure see [29] and [18].

5.0 VISUALIZATION AND COMPARISON OF MATRIX SPACES

[2] A gyroscope is a prototypic example of gyrator. Gyrators are frequently used for physical systems modeling [22].

[3] An electrical analogy for this situation is as follows: inverses of resistance, capacitive reactance, and inductive reactance are special cases of an RLC network admittance. For the lack of a better term, the word admittance is used both in electrical domain and mechanical domain.

[4] An exact necessary and sufficient condition exists for realizability, but testing a matrix for realizability using this condition is computationally intractable [5]

We have observed that *any* dominant accommodation matrix may be synthesized in a methodical way. Since we cannot guarantee the synthesis of any non-dominant matrix, dominant matrices are considered, in a conservative way, to be the largest class of matrices a passive resistive network may adopt. To obtain a measure of how large the range of dominant matrices is, we will compare it with the range of positive semi-definite (PSD) matrices. The largest class of matrices one might hope to synthesize with a gyrator-less passive system in a quasi-static interaction is the class of PSD matrices.

5.1 Space of Positive Semidefinite Matrices

How large is the space occupied by the PSD matrices? Let us associate each accommodation matrix with a point in $R^{n(n+1)/2}$. For instance, any 2×2 symmetric matrix of the form $\begin{bmatrix} a & b \\ b & c \end{bmatrix}$ can be represented as a point (a, b, c) in R^3. A set of accommodation matrices, therefore, becomes a volume in that space.

We now make the following observation:

Result 1. *The space of all n×n symmetric PSD matrices is a hypercone in an n(n+1)/2 dimensional space. Strictly PSD matrices (matrices with at least one zero eigenvalue) lie on the boundary of the cone and the inner volume represents the space of positive definite matrices.*

The representation of PSD matrices as a cone is well-known in linear algebra. One may refer to Hill and Waters [13] for mathematical treatment of cones of PSD and dominant matrices. We take a slightly different approach suitable for our purpose as described in the next few paragraphs.

Since any non-negative multiple of a PSD matrix is itself a PSD matrix, the space of these matrices may be represented by an infinite cone. Since a PSD matrix is symmetric, and an $n \times n$ symmetric matrix has only $n(n+1)/2$ distinct elements, we imagine, for simplicity, a cone in an $n(n+1)/2$ -dimensional space.

A sufficient condition for a symmetric matrix to be positive semidefinite is that the determinant of each of its principal submatrices is non-negative. Applying this condition on a general symmetric matrix leads to a set of inequalities that must be satisfied by the elements of the matrix. For the 2×2 matrix shown above, the inequalities are

$$a \geq 0 \qquad \text{and} \qquad ac - b^2 \geq 0 \,. \tag{2}$$

It can be shown that the complete set of (a, b, c) that satisfies the above conditions lie within an elliptical cone – a cone with elliptical cross-section touching the a-axis and the c-axis. The apex of the cone is at the origin (0, 0, 0). See Fig. 3 for a sketch of this cone, truncated by an inclined plane in order to reveal its elliptical cross section. It is to be understood here that any point inside this cone represents a positive definite matrix (which is also positive *semi*-definite by definition), and any point on the boundary of this volume represents a *strictly* PSD matrix, one of whose eigenvalues is always zero.

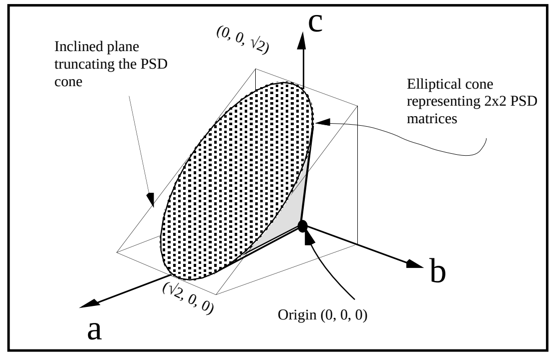

Fig. 3. Here we show the elliptical cone representing the set of all 2×2 positive semidefinite matrices as truncated by a 45° inclined plane.

5.2 Dominant Basis Matrices

In order to identify the characteristic volume of dominant matrices in a matrix space, we have found it useful to create a set of dominant *basis matrices* spanning the space of dominant matrices. We state the following result:

Result 2. *Any n×n dominant matrix can be expressed as a non-negative linear combination of a basis set of n^2 dominant matrices. These we call the **dominant basis matrices**.*

These basis matrices can be compared with the basis vectors spanning a linear vector space where any arbitrary vector can be expressed as a linear combination (positive, and negative) of the basis vectors.

According to the above result, any $n \times n$ dominant accommodation matrix $\boldsymbol{A}$ may be expressed as:

$$\boldsymbol{A} = \sum_{i=1}^{n^2} \alpha_i \boldsymbol{A_i} \tag{3}$$

where the α_i are non-negative scalar coefficients and the $\boldsymbol{A_i}$ are dominant basis matrices.

As an example, let us take a general 3×3 dominant matrix $\boldsymbol{A}$ of the form

$$\begin{bmatrix} a & b & d \\ b & c & e \\ d & e & f \end{bmatrix} .$$

Dominance requires $a \geq |b| + |d|$, $c \geq |b| + |e|$, and $f \geq |d| + |e|$.

Matrix **A** may be expressed (as in Equation 3) as a linear combination of the dominant basis matrices $\boldsymbol{A_{1...9}}$,

$$\boldsymbol{A_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{A_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{A_3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
$$\boldsymbol{A_4} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{A_5} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ \boldsymbol{A_6} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \tag{4}$$
$$\boldsymbol{A_7} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix},\ \boldsymbol{A_8} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix},\ \boldsymbol{A_9} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix},$$

along with the non-negative coefficients:

$$\alpha_1 = a - (|b| + |c|)\ , \quad \alpha_4 = \frac{|b| + b}{2}\ , \quad \alpha_7 = \frac{|c| - c}{2}\ ,$$
$$\alpha_2 = d - (|b| + |e|)\ , \quad \alpha_5 = \frac{|b| - b}{2}\ , \quad \alpha_8 = \frac{|e| + e}{2}\ , \quad (5)$$
$$\alpha_3 = f - (|c| + |e|)\ , \quad \alpha_6 = \frac{|c| + c}{2}\ , \quad \alpha_9 = \frac{|e| - e}{2}\ .$$

We note here that each of the dominant basis matrices is positive semidefinite of rank 1, i.e. each has only one positive eigenvalue. Also we observe that although n^2 dominant basis matrices are necessary to represent the full range of $n \times n$ dominant matrices, for any given $n \times n$ matrix we need only a set of $n(n+1)/2$ basis matrices. This is so because, depending on the sign of the off-diagonal elements, some of the α_i become zero.

5.3 Characteristic Volume of Dominant Matrices

Characterization of the volume of dominant matrices in a matrix space becomes easier if we make use of the dominant basis matrices. We make the following observation:

Result 3. *The space of all $n \times n$ dominant matrices represents a polyhedral convex cone (PCC) in a $\frac{n(n+1)}{2}$ dimensional space. The cone has n^2 edges, each corresponding to one of the dominant basis matrices. The edges of the PCC coincide with the boundary of the cone representing PSD matrices.*

Since any non-negative multiple of a dominant matrix is a dominant matrix itself, it is clear that the characteristic volume should be a cone. Each of the n^2 dominant basis matrices (when treated as a point in a matrix space as was done in section 6.1), together with its non-negative multiples, generates a ray giving rise to one edge of the PCC. Thus the PCC has n^2 edges.

For example, for 2×2 dominant matrices we have the following four dominant basis matrices

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \text{ and } \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} .$$

Each of these basis matrices corresponds to a point along one of the rays defining the edge of the PCC, namely the points (1, 1, 1), (1, -1, 1), (1, 0, 0), and (0, 0, 1). See Fig. 4 for a sketch of the PCC representing the characteristic volume of 2×2 dominant matrices and the four rays generating the dominant PCC.

We recall that each dominant basis matrix is a strictly PSD matrix, as are its non-negative multiples. Therefore the edges of the PCC coincide with the boundary of the PSD cone. At this point, we have a way of visualizing that "all dominant matrices are PSD but the contrary is not true". This we do by observing that the dominant PCC lies within the bounds of the PSD cone. In Fig. 5 dominant PCC and PSD cones are superposed in order to show this.

One way to compare ranges of dominant matrices and PSD matrices is to compare the volumes of their respective cones. Since cones have one axis along which no structural detail is changed (only the size of the cross section is reduced or amplified) one may consider comparing similar sections of different cones as a means of comparing the volumes of the cones. This is reasonable since volumes of cones of same height are proportional to their 'footprints' on the same plane. For convenience, we will consider the intersection of each cone with a plane that is perpendicular to the line of symmetry of the cone and at unit distance from the origin. We now have the following result which is derived from Results 1 and 3.

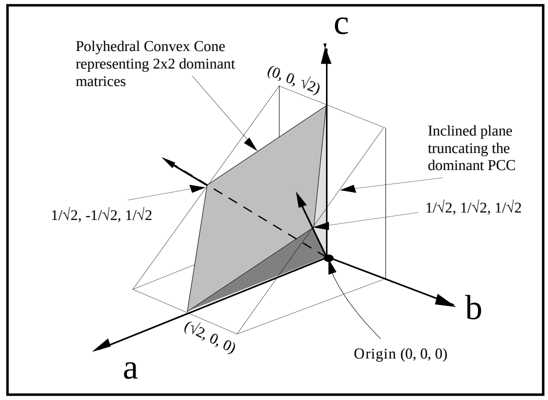

Fig. 4. Here we show the polyhedral cone representing the set of all 2×2 dominant matrices as truncated by a 45° inclined plane.

Result 4. *The PCC representing the set of dominant matrices when intersected with a hyperplane gives rise to a convex polytope. For $n \times n$ PSD matrices the hypercone and the truncating plane are of dimensions \F(n(n+1),2) and [$\frac{n(n+1)}{2}$ - 1] respectively. The polytope has n^2 vertices corresponding to the n^2 dominant basis matrices. These vertices lie on the boundary of the intersection of PSD cone and hyperplane.*

Therefore, the set of all 6×6 PSD matrices represent a cone in a 21-dimensional space (21 is the number of distinct elements in a 6×6 symmetric matrix). The intersection of the cone and a 20-dimensional hyperplane perpendicular to its axis gives rise to a 20-dimensional volume.

The set of all 6×6 dominant matrices represents a PCC in a 21-dimensional space. The PCC has 36 edges. If we intersect the PCC with a 20-dimensional hyperplane, we get a 20-dimensional polytope with 36 vertices. Since each of the dominant basis matrices is PSD, we can say that the vertices of the dominant polytope lie on the boundary of the characteristic volume for PSD matrices.

For 2×2 matrices, looking at the intersection of the 3-dimensional characteristic cones with the 2-dimensional inclined planes we notice that the volume of PSD matrices is represented by an ellipse (loosely referred to as the *PSD* ellipse*)* and that of the dominant matrices is represented by a quadrilateral (loosely referred to as the *dominant quadrilateral*) inscribed in the ellipse. Fig. 5 shows the PSD ellipse and the dominant quadrilateral as seen on the inclined plane.

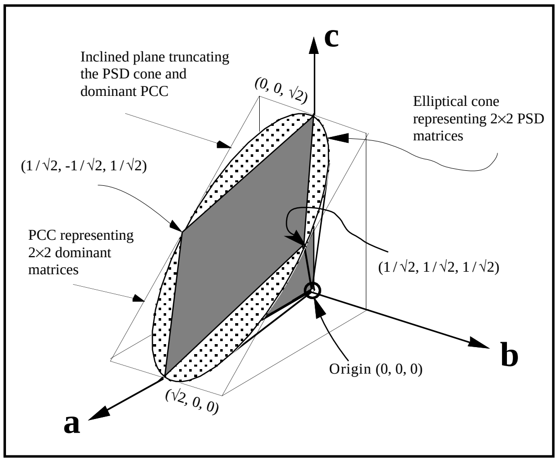

Fig. 5. This is a superposition of Figs 3 and 4. This shows that the dominant PCC is completely inside the PSD cone implying the fact that dominant matrices are a proper subset of PSD matrices.

All of the above discussion has been in terms of the joint-space accommodation matrix (or the hydraulic conductance matrix) of the hydraulic network interconnecting the ports of the hydraulic cylinders in a passive device.

6.0 JOINT-SPACE, TASK-SPACE, AND CONGRUENCE TRANSFORMATION

We need to distinguish between the joint-space and the task-space of a manipulator and describe how accommodation matrices are mapped between these spaces.

6.1 Theoretical Results

A passive robotic device (an assembly wrist, say) can be thought of in the usual robotic terms as several joints or actuators coupled in a suitable way to give a desired spatial structure. Joint-space describes the so-called actuator variables, such as position, velocity, acceleration, force, etc. of the actuators. The dimensionality of joint-space is equal to the number of passive joints (hydraulic cylinders, say) present in the wrist. Task-space, on the other hand, can be defined as the 6-dimensional space describing the forces on or motions of the manipulated workpiece. The origin of the task-space coordinate frame is typically situated in the workpiece.

The accommodation matrices suitable for a given task are most conveniently described in terms of the task-space variables of a robot. For example, Schimmels and Peshkin describe the procedure for generating task-space accommodation matrices which are useful for automated force-guided insertion of a workpart into a fixture [26]. However, the synthesis procedure for realizing the matrix for a particular wrist is most naturally undertaken in the joint-space as the theoretical results (Sections 5.0) are available in this space. In order to make network theory results more useful for assembly tasks, we should project these results into task-space and re-interpret them in terms of task-space variables. Applying transformations to velocity and force components, we get the following relationships:

$$\boldsymbol{A}_t = \boldsymbol{J}\,\boldsymbol{A}_a\,\boldsymbol{J}^T \qquad (6)$$

Where $\boldsymbol{A}_a$ and $\boldsymbol{A}_t$ are the accommodation matrices in the joint-space and in the task-space respectively. $\boldsymbol{J}$ is the Jacobian matrix of the wrist.

The transformation of accommodation matrices from joint-space to task-space is an example of *congruence transformation* and as shown in Equation 6, it involves the manipulator (wrist) Jacobian. Thus the available task-space matrices are dependent on the manipulator configuration.

The congruence transformation of admittance-type matrices (compliance, accommodation etc.,) and impedance-type matrices (stiffness, damping etc.) from one space to another has been dealt in the literature before. For a representative sample of literature, see [12, 23]. A somewhat similar problem is encountered in the stiffness analysis of robotic hands. See for example [16, 24].

We now describe the following result about congruence transformation on PSD matrices:

Result 5. *The cone representing PSD matrices in the matrix space is invariant under congruence transformation. i.e. no matter what Jacobian is used, the joint-space PSD cone and the task-space PSD cone are identical.*

This result is a consequence of well-known *Sylvester's Law of Inertia* [15]. Intuitively, since positive semidefiniteness is associated with basic requirements of passivity of a network, a device that is passive in joint-space is expected to remain passive in task-space also.

How do the joint-space dominant matrices transform to task-space? In other words, if we know which joint-space accommodation matrices are synthesizable what is the range of accessible task-space matrices? We have found the following result:

Result 6. *The manipulator Jacobian (which is a function of the manipulator's configuration) maps the joint-space dominant PCC to a task-space PCC. The task-space PCC (which is not in general dominant) represents the volume of synthesizable task-space accommodation matrices. The edges of the task-space PCC coincide with the boundary of the PSD cone. Consequently, the vertices of the task-space polytope coincide with the boundary of the intersection of the PSD cone and a hyperplane*[5] .

Since each dominant basis matrix is PSD in joint-space, they remain PSD in task-space. The edges of the synthesizable task-space PCC, corresponding to the dominant PCC in joint-space, therefore still coincide with the boundary of the task-space PSD cone. Consequently, the vertices of the polytope obtained by truncating the synthesizable task-space PCC with a hyperplane are on the boundary of the intersection of the task-space PSD cone with a hyperplane. Here we note that since dominance is not a condition that is preserved under congruence transformation, the synthesizable matrices in the task-space are not necessarily dominant. Therefore, the synthesizable PCC in joint-space and in task-space do not, in general, have the same shape. The Jacobian of the manipulator, which is related to the manipulator configuration, maps the joint-space PCC to the task-space PCC.

[5]The polytope is obtained by intersecting the task space PCC with a hyperplane . The PSD cone is similarly intersected by a hyperplane to give us a convex volume.

6.2 Examples

We show the nature of joint-space to task-space transformation with an illustrative example. We consider a simple two-cylinder parallel mechanism in Fig. 6 (inset Fig. 6) having a two-dimensional task-space. In the figure, PSD ellipse, joint-space dominant quadrilateral, and corresponding task-space quadrilaterals for a given posture are shown.

The areas of ellipses and quadrilaterals can be compared to obtain a relative measure of the range of the dominant matrices to that of the PSD matrices. We might even want to determine the posture of the manipulator that gives us the maximum range of synthesizable matrices in the task-space. For the 2-dof example, the area of the PSD ellipse[6] is $\frac{\pi}{\sqrt{2}}$. The area of the joint-space dominant quadrilateral is $\sqrt{2}$. It is interesting to note that the dominant quadrilateral possesses the maximum area of any quadrilateral inscribed in the PSD ellipse.

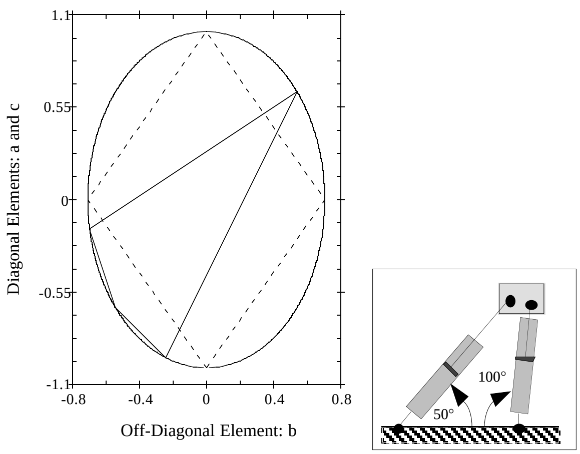

Fig. 6. Synthesizable accommodation matrices in the task-space for different configurations of a 2 dof parallel planar manipulator is given by the solid quadrilaterals above. PSD ellipse and dominant quadrilateral (dashed) are shown for comparison.

The joint-space dominant quadrilateral and the task-space quadrilateral are identical when the Jacobian is an identity matrix. For both serial and parallel manipulators (2 dof), this happens when the joint angles are 0° and 90° apart. Since there are an infinite number of quadrilaterals of maximum area inscribed in an ellipse, an infinite number of manipulator postures give us the maximum range of synthesizable accommodation matrices. For instance, for the serial manipulator, whenever the joint angles are 90° apart we have maximum range of accommodation matrices. Fig. 7 shows such a posture for the serial manipulator that give us the maximum area.

The size of synthesizable PCC reduces continuously as a manipulator approaches singularity. For a 2 dof manipulator the volume of task space PCC reduces to zero at singularity.

[6]The area really depends on the location and orientation of the truncating plane which in this case is perpendicular to the axis of the cone and positioned at unit distance from the origin on the axis.

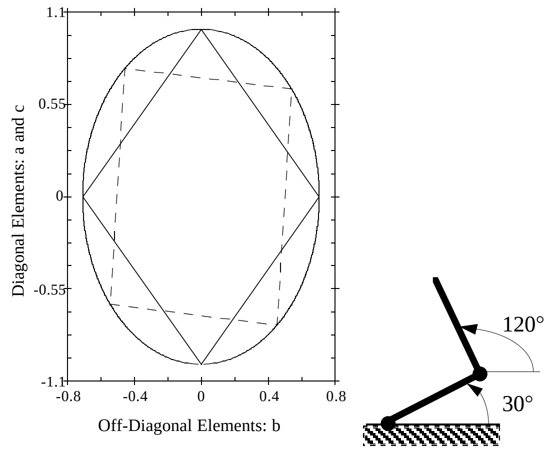

Fig. 7 Similar to Fig. 6. The dashed quadrilaterals in this Fig. give the maximum area, an area equal to the dominant quadrilateral.

In the previous examples, we obtained a class of matrices in the task-space that are not dominant matrices. This can be explained by the fact that the manipulator links are mechanical equivalents of electrical transformers. Therefore, when the hydraulic network is viewed from task-space, it is a network of resistances *and* transformers, having the capability of possessing any PSD matrix, given the full flexibility of transformer parameters (which for a manipulator are functions of the link lengths and the joint angles). To take advantage of this we would have to adjust the manipulator design. This also justifies our selection of PSD matrices as the standard class of matrices against which dominant matrices are compared. However since we are considering manipulator of a particular geometry, which is equivalent to a network of transformers with fixed parameters, we are not obtaining the full range of PSD matrices in task-space.

7.0 SUMMARY

The thesis of this paper is that a passive robotic wrist, of fixed mechanical design, can be programmed to execute a wide range of force control laws useful in automated assembly. In this paper, we conducted a systematic study to characterize the range of control laws (given by accommodation matrices) implementable by passive programmable hydraulic devices. We used electrical network theory results to identify the accommodation matrices which are available in the joint-space of the wrist. We then projected these matrices in the task-space and compared the range of task-space matrices with a known class of matrices in an attempt to quantify the usefulness of passive devices.

For simplicity, we considered hydraulic networks consisting of tunable dampers. A network of this kind may be imagined to interconnect the cylinder ports of a passive Stewart platform type robotic wrist and lets one program a desired accommodation matrix. We show that a broad range of accommodation matrices may be synthesized by passive devices with programmable parameters.

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