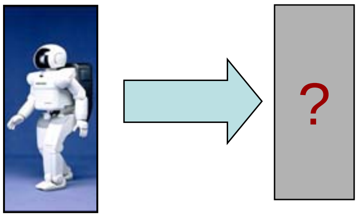
Kinematic and dynamic analogies between planar biped robots and the Reaction Mass Pendulum (RMP) model

Honda Research Institute USA, Inc.

Ambarish Goswami
Honda Research Institute USA Inc., Mountain View, CA
agoswami@honda-ri.com

HONDA The Power of Dreams

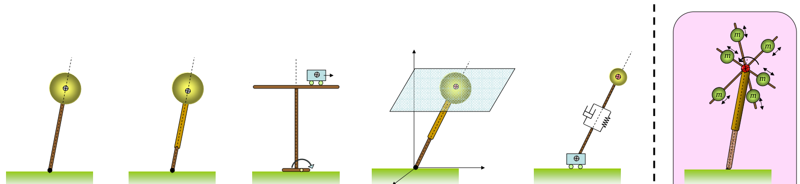
OBJECTIVE: To develop a reduced model of humanoid robot for analysis and control of balance and gait, and especially to model robot's angular momentum. We have proposed the Reaction Mass Pendulum (RMP) model for this purpose and demonstrated the kinematic mapping between a robot and a 3D RMP (*Lee and Goswami, ICRA 2007*). Here we present an in-depth analysis of the dynamics of planar robots and 2D RMP.



Humanoid dynamics is complex (high dof, nonlinearity, 3D rotational motion, external contact constraints etc.)

Ideal: To develop a "*minimal but complete*" model to capture humanoid dynamics for efficient formulation of gait and balance control

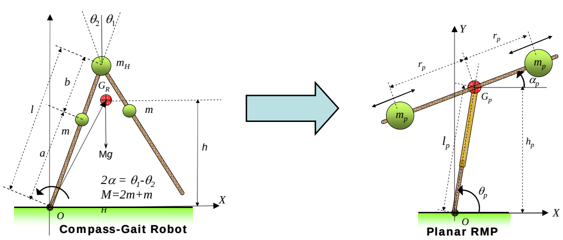
However, a "*minimal but complete*" model is application-specific and elusive. One may instead settle for "*reduced but sufficient*" model. Many useful reduced models, although without an angular momentum component, have been benefiting the study of human and humanoid gait and balance:



Rigid Inverted Pendulum | Telescopic Inverted Pendulum | Cart-Table (*Hirukawa et al.*) | 2D and 3D Linear Inverted Pendulum (*Kajita et al.*) | Variable Impedance Inverted Pendulum (*Sugihara & Nakamura*) | 3D Reaction Mass Pendulum (RMP) (*Lee & Goswami*)

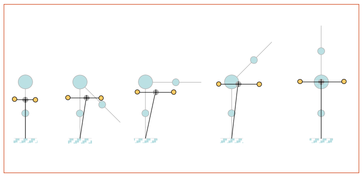
Model features and kinematic equivalence:

Here we contrast and compare the full kinematics and dynamics of a planar compass-gait robot and the planar version of RMP.



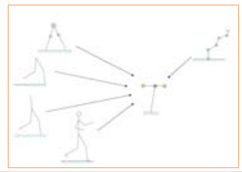
Compass-Gait Robot | Planar RMP

Kinematic equivalence between compass-gait robot and planar RMP.



Robot and RMP have identical CoP, CoM, linear mass and Centroidal Composite Rigid Body Inertia (CCRBI). Consequently, they have identical potential energy.

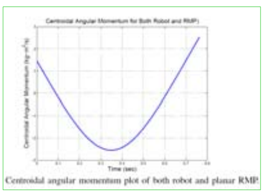
Generality of RMP Model: A broad variety of planar mechanisms including humanoid robots in different configurations, and even fixed-base robots can be mapped to its corresponding RMP.



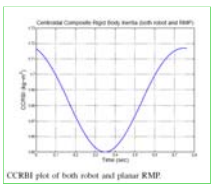
Dynamic Equivalence:

To obtain dynamic equivalence, we could derive a generalized velocity mapping between the robot and the RMP that preserves kinetic energy. We instead use a velocity mapping that *preserves the centroidal angular momentum between the robot and the RMP.*

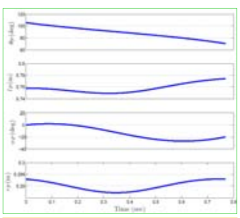
All gait data for compass gait robot was generated by Dr. Fumihiko Asano, RIKEN.



Centroidal angular momentum of robot and Planar RMP

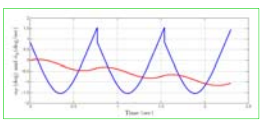


CCRBI of robot and Planar RMP

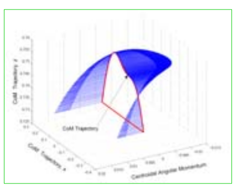


Generalized coordinates of RMP.

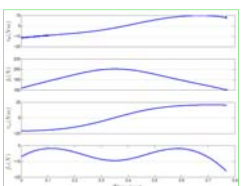
For a given centroidal angular momentum, the ***average angular velocity*** of a multi-body system is that which its CCRBI must instantaneously possess.



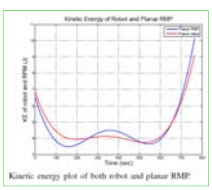
Average angular velocity (blue) and rotational angle (red) of RMP.



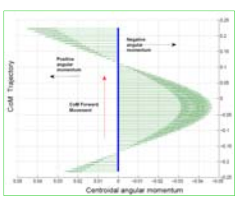
Centroidal angular momentum shown on 3D CoM trajectory of compass gait robot



Generalized forces of RMP.



Centroidal Angular momentum preserving mapping does not preserve kinetic energy



Top view of the above figure to clearly show direction change of centroidal angular momentum

A principle of torque mapping:

Centroidal angular momentum preserving torque mapping between robot and RMP:

What is the torque mapping between a robot and its RMP such that their centroidal momentum are equal?

Joint torque and centroidal angular momentum rate change are related by an affine mapping

$h_G = A_G(q)\dot{q}$

$\dot{h}_G = A_G\ddot{q} + \dot{A}_G\dot{q}$

$\tau = H(q)\ddot{q} + C(q,\dot{q})\dot{q} + \tau_g(q)$

$\ddot{q} = H^{-1}(\tau - C\dot{q} - \tau_g)$

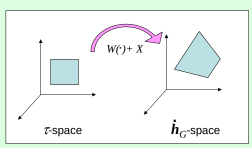
$\dot{h}_G = A_G H^{-1}(\tau - C\dot{q} - \tau_g) + \dot{A}_G\dot{q}$

$\dot{h}_G = \mathcal{W}\tau + \mathcal{X}$

Torque sensitivity matrix

where, $\mathcal{W} = A_G H^{-1}$

$\mathcal{X} = (\dot{A}_G - \mathcal{W}C)\dot{q} - \mathcal{W}\tau_g$



$\|\tau\|_2 \rightarrow \dot{h}_G$

"Balance ellipsoid"

Let us set:

$\dot{h}_{G_R} = \dot{h}_{G_P}$ → $\mathcal{W}_R\tau_R + \mathcal{X}_R = \mathcal{W}_P\tau_P + \mathcal{X}_P$

$\tau_R = \mathcal{W}_R^{+}[\mathcal{W}_P\tau_P + (\mathcal{X}_P - \mathcal{X}_R)]$

Conclusions:

- *Conceptually simple inverted pendulum models are useful for the study of biped balance.*
- *Full kinematic and dynamic analogy between planar biped and planar RMPare explored*
- *One cannot get both angular momentum and kinetic energy equivalence simultaneously.*
- *Angular momentum mapping leads to non-cyclic RMP motion.*
- *Obtained a joint torque map that preserves angular momentum between robot and RMP.*

Future Work:

- *RMP based movement generation for compass gait. Exploitation of angular momentum control.*
- *Torque mapping and dynamic equivalence for 3D humanoid*