PASSIVE ROBOTICS: AN EXPLORATION OF MECHANICAL COMPUTATION

Ambarish Goswami
Michael A. Peshkin
James Edward Colgate

Mechanical Engineering Dept.
Northwestern University
Evanston, IL 60208

Abstract

We invite the reader to think of a passive wrist as a mechanical computer. The wrist computes a particular motion in response to every applied force, and this defines its control law. Suppose that the *design* of a wrist (the geometric layout of mechanical elements -- springs, hydraulic cylinders, dampers, and so on -- which compose it) is held fixed. We can "program" the wrist by changing the parameters (for example, the spring stiffnesses) of some of these elements. What is the range of control laws such a wrist can execute?

The thesis of this paper is that a passive wrist, of fixed design, can be programmed to execute a wide range of useful control laws. We consider in particular wrists whose actuators are unpowered hydraulic cylinders, the ports of which are coupled to one another via variable-conductance constrictions. By selection of these conductances the wrist is programmed, much as an analog computer is programmed. We characterize mathematically the range of control laws such a device can compute.

Motivation

Positional errors of mating parts in assembly operations give rise to characteristic forces. In principle one may be able to determine the needed corrective motion from a knowledge of the contact forces alone. We have an ongoing interest in the idea of "programming" a manipulator's compliance so that the forces which characterize every error naturally result in the motion which corrects the error. In related work [9] we have detailed how the accommodation matrix of a manipulator (similar to a compliance matrix) may be synthesized so that it is error-corrective for a particular assembly operation.

If we can synthesize an accommodation matrix which is appropriate for a given task, we must then confer that behavior on a manipulator either by active or by passive means. If implemented actively, stability and speed of operation may be a limiting factor [13], but we have great flexibility in programming the desired accommodation matrix. Passive devices can adopt a more limited class of accommodation matrices. By virtue of passivity, however, stability is guaranteed at all frequencies [5], while the local analog computation of motion can result in high bandwidth. For this reason, it would be preferable to implement the desired behavior passively, i.e. to arrange the natural mechanical compliant behavior of a manipulator so that it matches the accommodation matrix we synthesized.

This work was supported by NSF grant DMC-8857854, Northwestern University, and a gift from IBM Corporation.

In this paper we characterize the range of accommodation matrices that can be implemented by a passive mechanical device. Some of our results apply to any passive wrist, and some to wrists whose components are restricted to hydraulic cylinders and dampers.

Sample of Results

Before addressing the general problem of creating a passive physical device which possesses any desired compliance, we give one specific result which can be immediately appreciated.

The RCC wrist [12] has proven its utility as a translator of assembly forces into corrective motions for a particular task: peg into chamfered hole. RCC possesses a "sweet spot" (a center of compliance) near which the tip of the peg must fall. Shown in Figure 1 is a Stewart platform wrist consisting of 3 parallel hydraulic actuators. We can change the compliance (strictly speaking, the accommodation matrix) of this device by selecting the hydraulic conductances of the interconnections of the ports of the cylinders. Over what range of space can the "sweet spot" be moved, simply by

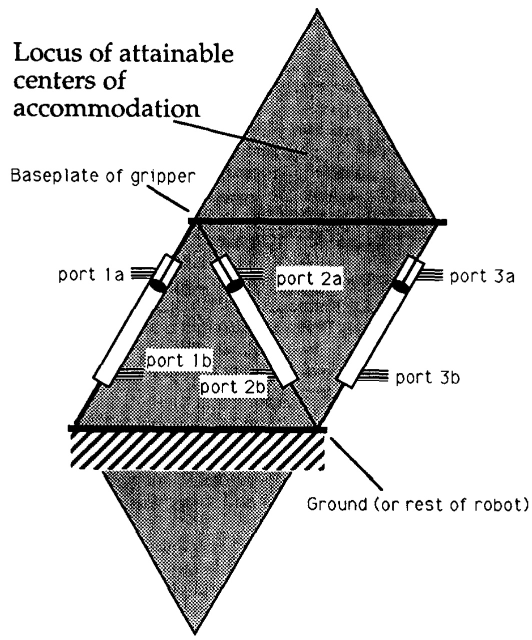

Figure 1. Locus of attainable centers of accommodation for a 3 DOF planar Stewart platform geometry wrist.

varying the conductances? The substantial region of attainable centers of accommodation shown in Figure 1 is an encouraging result[2], holding promise that a reprogrammable wrist may be able to adopt a powerfully broad set of accommodation matrices. (See for comparison [6] for an example of a programmable passive wrist the compliance center of which can be be moved along a line.)

We wish to stress that accommodation matrices of interest to us are by no means restricted to those which possess a "center" (sweet spot). In fact, our recent work on the synthesis of error-corrective accommodation matrices [10] often results in accommodation matrices which do not have a center of accommodation. The set of all accommodation matrices which are attainable by reprogramming a wrist of fixed design cannot, unfortunately, be described so simply and graphically as the limited subclass discussed above. Mathematical description is necessary, and in this paper we address that problem.

Compliance, Accommodation, and Inverse-Inertia Matrices

The term "compliance" is often used in a loose sense to describe the particular way (characterized by ensuing displacement, velocity, or acceleration) in which a device responds to imposed forces. For instance, a system of spring responds to imposed forces by means of characteristic displacements, a system of dampers by characteristic velocities, and a system of masses by characteristic accelerations. More strictly, a compliance matrix describes a *generalized spring*, an accommodation matrix describes a *generalized damper*, and an inverse-inertia matrix describes a *generalized mass.*

The input-output relationship of a linear system can be explicitly written as a *transfer function.* For a multi-input multi-output system we write a *transfer matrix* that maps the inputs into the outputs (both expressed as vectors). A matrix that maps the forces imposed on a system into the resulting velocities is called an *admittance matrix.* In its most general form, the elements of this matrix are functions of complex frequency (in Laplace domain) and can describe the behavior of all passive elements (springs, dampers, masses etc.) as well as any active behavior, so long as it is linear. Thus, the accommodation matrix, compliance matrix, and inverse-inertia matrix are all special cases of admittance matrices. They characterize behaviors of networks composed of elements of single kinds.

Suppose a workpiece is held by the compliant wrist of a robot and is moving with a nominal velocity $\mathbf{V}_0$ in the absence of any assembly forces. If the admittance matrix of the manipulator is $\mathcal{A}$, then the resultant velocity of the workpiece $\mathbf{V}$ due to both the nominal velocity of the manipulator, and its response to assembly forces $\mathbf{F}$, will be

$$\mathbf{V} = \mathbf{V}_0 + \mathcal{A}\mathbf{F} \qquad (1)$$

Note that $\mathbf{V}$, $\mathbf{V}_0$ and $\mathbf{F}$ are 6-vectors (translational and rotational velocities, or forces and torques).

Although our discussions will be based on the most general form of admittance matrix we will keep a special interest on the matrices that characterize generalized dampers. The mechanical components needed to create a generalized damper are hydraulic cylinders and hydraulic constrictions (or "resistors"). As a practical matter programmable hydraulic resistors are easier to build than programmable springs, and it is hard to imagine what a programmable mass might be. We will concentrate on the realization of generalized dampers as passive physical devices.

[2] An analogous result applies for the 6 DOF wrist.

Scope of This Paper

The questions we address in this paper are

1. What are the restrictions on a desired admittance matrix, such that it will be possible to realize it as a passive physical device?

2. How can these devices be systematically designed?

3. Is it possible to design a single wrist, having some user selectable (or, "reprogrammable") parameters, such as spring stiffnesses or damping coefficients, so that any desired admittance matrix can be assumed by this wrist, simply by reprogramming this wrist and without changing its geometry, dimensions, and topology?

The problem of determining the properties that an admittance matrix must have so that it can be assumed by a *particular* wrist is called the *realizability problem*, in analogy to electrical network theory. It turns out that there are realizable admittance matrices (i.e., admittance matrices that a wrist can be programmed to possess) for which we nevertheless cannot *algorithmically* synthesize a wrist. By the *synthesis problem* we will refer to the problem of classifying realizable matrices that are algorithmically synthesizable.

Passive Devices

Bond graph theory draws parallels among elements in different physical domains - electrical, mechanical, hydraulic etc. [7, 8]. According to this theory, interactions of all physical systems occur through a transfer of power. Transfer of power can be most conveniently expressed either by generalized power variables called *effort* and *flow* or by generalized energy variables called *momentum* and *displacement*. Force, voltage, and pressure represent different forms of *effort*; velocity, electrical current, and volume flow are different forms of *flow*. Mechanical momentum and flux linkage are *momenta*. Linear or angular displacements, volume and charge are examples of *displacement*.

We have available to us all passive components to be used in a wrist. Bond graph theory puts electrical resistors, mechanical dampers and hydraulic porous plugs in a single class of elements that maps an effort into a flow. This is called the class of one-port *resistors*. One-port *capacitors* are those elements that have a static relationship between an effort and a displacement. Electrical capacitors, mechanical springs, air bladders etc. fall into this class. Similarly, one-port *inertias* relate a momentum to a flow. Examples are electrical inductors, masses etc. There are also mechanical and hydraulic analogues to the two-port passive elements (*transformers* and *gyrators*) but we shall give these less attention here.

Actuator Space and Task Space

We must make the distinction between *task space* and *actuator space*. Task space {t} is the 6-dimensional space describing the forces on or motions of the grasped object.

We imagine a coordinate frame, the *task frame*, which is used to describe the task space. The origin of the task frame is typically situated on the grasped object. As an example, in peg-in-hole assembly the task frame is fixed with respect to the peg. (See Figure 2)

The wrist itself can be thought of in the usual robotic terms as several "joints" or actuators coupled in a suitable way to give a desired spatial structure. Actuator space {a} describes the so called actuator variables, such as position, velocity, acceleration, force etc. The dimension of the space is equal to the number of actuators present in the wrist. A Jacobian transforms task space variables (force, velocity etc.) to actuator space variables.

The required force-velocity relationships (e.g., admittance matrices) for a given task are most conveniently described in its task space. However, the realization procedure is associated with the changing of parameters of the elements that constitute the wrist and it is the actuator space in which we change those parameters. Therefore, the most natural way to study the realization of a task-space admittance matrix ($\mathcal{A}_t$) is to transform it into the actuator space admittance matrix ($\mathcal{A}_a$) and to find out whether it is realizable. It can be shown that by applying transformations to velocity and force components in the task space, we get the following relationship

$$\mathbf{A}_a = {}^{a}_{t}\mathbf{J}\ \mathcal{A}_t\ {}^{a}_{t}\mathbf{J}^T \qquad (2)$$

where ${}^{a}_{t}\mathbf{J}$ represents the Jacobian which transform velocities from task space {t} to actuator space {a}. The transpose of this Jacobian relates forces in those spaces.

Electrical Analogy

According to bond-graph theory, there are equivalences among physical networks of different kinds. For example, a network consisting of mechanical elements has an equivalent electrical counterpart. The behavior of one is directly related to the behavior of the other. Much is known about the analysis and synthesis of passive electrical networks and quite a few useful results are available to us. We will make use of those results here.

As an example, we describe how a hydraulic network can be compared to an electrical network. In a hydraulic network, the force on a unit area piston is the pressure difference P across the piston. P can be compared to a voltage source in an electrical network which simply creates a potential difference (E). The piston velocity (V) is analogous to electric current (I). Also, we can draw an analogy between hydraulic conductance (Y) and electrical conductance (G) of a resistor in respective networks. Electrical conductance is defined as the amount of current that passes through a conductor when unit voltage is applied across it, and is the inverse of resistance. Equivalently, hydraulic conductance (of a constriction, say) is defined as the flow rate that is allowed through when it is subjected to unit pressure difference.

An example

To demonstrate the electrical analogy, an example is given. For simplicity, we are considering a network consisting of dampers only. The objective here is to calculate the force-velocity relationship (an accommodation matrix, in this case) for the hydraulic network of Figure 3a. The equivalent electrical circuit is shown in Figure 3b.

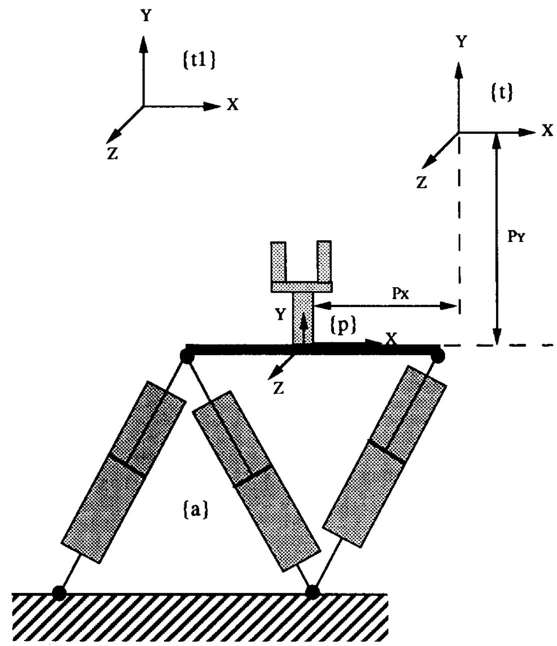

Figure 2. Relation of Task Space and Actuator Space

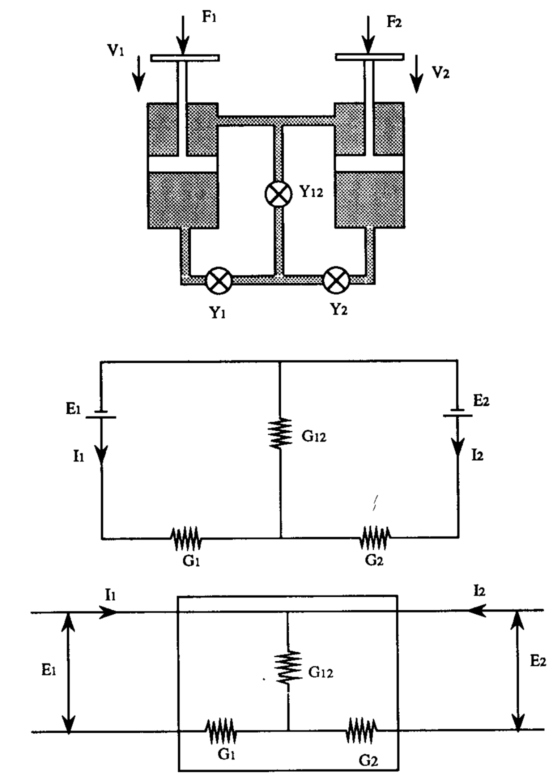

Figure 3. A Hydraulic Network (a), its Electrical Equivalent (b), and the Corresponding Two-Port Representation (c).

A description of a purely resistive network includes equations (derived from Kirchhoff's laws, for instance) involving constant resistances and the variables (voltages and currents) of different branches of the network. There are several ways in which the variables of a network can be expressed. Here, we are interested in the voltages and currents only at certain locations or *ports* of a network, and we call them *port variables*. (The ports will coincide with the actuators.) The rest of the variables are called the *internal variables* of a network. We eliminate the internal variables and obtain a set of relationships exclusively among port variables. The port variables are chosen in such a way that sources exist only at the ports. This makes the rest of the network passive.

The networks whose external behaviors are expressed in terms of their port variables are classified in terms of the number of ports and are, in general, called *passive n-port networks*.

One way to describe a passive resistive n-port network is to write the port voltages and port currents as two n-dimensional vectors, E and I. These two vectors are related by a *transfer matrix* of the network. The transfer matrix can be an admittance matrix G, with I=GE, or an *impedance matrix* Z, with E=ZI, depending on what we choose to be the input to the system.

Figure 3b can be suitably redrawn as a two-port network as shown in Figure 3c. With the help of simple electrical analysis we can come up with the following relationships between applied potential differences and induced currents at the ports,

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} G_1(G_2 + G_{12})/G & -G_1G_2/G \\ -G_1G_2/G & G_2(G_1 + G_{12})/G \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \qquad (3)$$

where

$$G = G_1 + G_2 + G_{12} . \qquad (4)$$

Equation 3 can be rewritten in a more compact form as

$$\mathbf{I} = \mathbf{G}\,\mathbf{E} \qquad (5)$$

where **I** is the current vector of the ports and **E** is the voltage vector at the ports. Matrix **G** is termed as the admittance matrix which describes the behavior of the network.

As it is that current and voltage are respectively equivalent to velocity and force in a hydraulic network, the input-output relationship for the network in Figure 3a can be similarly expressed as,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_1(Y_2 + Y_{12})/Y & -Y_1Y_2/Y \\ -Y_1Y_2/Y & Y_2(Y_1 + Y_{12})/Y \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \qquad (6)$$

where

$$Y = Y_1 + Y_2 + Y_{12} . \qquad (7)$$

or equivalently as,

$$\mathbf{V} = \mathcal{A}\,\mathbf{F} \qquad (8)$$

This completes the demonstration of the equivalence between an electrical network and a hydraulic network.

Consequences of Passivity

Here we describe how the fundamental physical restriction of *passivity* on a wrist translates into mathematical restrictions on the set of admittance matrices which are, in principle, realizable. We give ourselves the full flexibility of considering wrists of *any possible design* in order to achieve the desired admittance matrix, and do not restrict ourselves to just reprogramming a wrist of a fixed design.

From first principles, we know that a passive wrist cannot, in any circumstance, generate power. This is the so called *passivity condition* [11]. The passivity condition translates to the statement that the complex admittance matrix $\mathcal{A}$ must be *positive real* [4] [11]. This means that for complex frequency $s = s + jw$, $\mathcal{A}$ must satisfy the following conditions

1. $\mathcal{A}(s) + \mathcal{A}^H(s)$ has no poles in the right half plane.
2. Any imaginary poles are simple and have positive real residue matrices.
3. $\mathcal{A}(jw) + \mathcal{A}^H(jw)$ is positive semi-definite Hermitian.

The superscript H denotes the Hermitian (complex conjugate transpose) of a matrix.
The three conditions described above constitute the necessary and sufficient condition for an accommodation matrix to be realizable with purely passive components.

If $\mathcal{A}$ is a real-valued admittance matrix (accommodation matrix), the above conditions simply mean that all eigenvalues must be non-negative.

Consequences of Limitation of Types of Passive Elements

Let us now consider the consequences of restricting the types of passive elements which may occur in our passive wrist, to eliminate some of the more esoteric elements.

If we remove gyrators from our device, the *actuator space* admittance matrix must be symmetric [1]. The elements of the Laplace domain admittance matrix are still functions of complex frequency 's'.

If we remove capacitors and inductors (or their mechanical analogues) from the possible range of available components, we are left with a *real* symmetric admittance matrix (Laplace domain admittance matrix independent of s.) This can be easily verified by noticing the fact that capacitors and inductors are the only frequency dependent passive elements. Gyrators, transformers, and resistors do not impose any phase-shift on the input signal, thus the terms in the admittance matrix do not contain imaginary parts.

The exclusion of transformers leaves us with a purely resistive circuit with the so called no-amplification property. When only one port of a resistive circuit is driven by a voltage or a current source, all voltages (if it is in open-circuit condition) and currents (if it is in short circuit condition) at the other ports in any load condition cannot be greater in modulus than the voltage and the current at the driver port. Cederbaum generalized the idea of no-amplification [2] and showed that the admittance matrix of a purely resistive circuit must be a paramount matrix. Civalleri [3] proved that a hypothetical network fulfilling no-amplification property but not paramountcy would surely amplify if some of its branches are cut away. So paramountcy is a more restrictive condition than no-amplification.

A real symmetric matrix is said to be paramount if any of its principal minors is not less than the absolute value of any other minor built upon the same rows (or columns) by replacing any number of columns (or rows).

Realizability and synthesizability of admittance matrices

Recall that an admittance matrix is *realizable* if a passive network exists with that admittance matrix. *Synthesizable* is a stronger condition meaning that the network can be created algorithmically.

A matrix is of course not realizable by a network of passive elements of certain types if it violates the condition of passivity, or if it falls outside the class of matrices appropriate for those types (described above). Let us now restrict ourselves to consideration of networks of one type of element: either of resistors, inductors, or capacitors. All realizable matrices will therefore be paramount. We will discuss this in terms of a network of resistors although identical results apply to networks entirely of capacitors or inductors. We will therfore be constructing accommodation matrices out of networks of resistors, while analogous results hold for constructing compliance matrices out of capacitors.

It has been found that the property of paramountcy does not guarantee realizability: paramountcy is a necessary but not a sufficient condition for realizability. There are presumably other restrictions on realizability that have not yet been identified [11]. Civalleri [3] proved that a hypothetical network fulfilling no-amplification property but not paramountcy would surely amplify if some of its branches are cut away. So paramountcy is a more restrictive condition than no-amplification.

A *sufficient* condition for an admittance matrix to be realizable is that it be *dominant* [11].

A real symmetric matrix is said to be dominant if each of its main diagonal elements is not less than the sum of the absolute values of all the other elements in the same row (or column).

It can be shown that all dominant matrices are paramount but the converse is not true. Further, there is an available algorithmic synthesis procedure for the realization of dominant matrices. In other words, dominant matrices are algorithmically synthesizable. As a summary of the above discussion we have:

All Matrices

- $\supset$ Positive Real Matrices (realizable with passive elements only)
  - $\supset$ Symmetric Positive Real Matrices (realizable with transformers and one-ports)
    - $\supset$ Symmetric Positive Semi-Definite Matrices (those realizable with transformers and resistors only)
      - $\supset$ Paramount Matrices (a necessary condition for realizability with resistors)
        - $\supset$ Dominant Matrices (a sufficient condition for algorithmic synthesizability with resistors)

There are in fact examples of networks whose admittance matrix is not dominant. There are examples of paramount matrices for which it can be proved that there is no realization.

Example of the range of matrices synthesizable for a fixed design

In order for a task space accommodation matrix to be algorithmically synthesizable, the corresponding actuator space matrix must be dominant. It is not possible to display graphically the entire class of synthesizable accommodation matrices for a given wrist design. Therefore we choose a very special subclass of these, specifically, the accommodation matrices which possess a center of accommodation at some point in space. Over what region of space can we place the accommodation center, purely be programming the network?

We imagine a coordinate frame, called the *platform frame*, within which we describe the origin of a task frame. In Figure 2 the platform frame is attached to the top plate of the Stewart platform. It should be understood here that the same task-space accommodation matrix in two different task frames (e.g. $\{t\}$ and $\{t_1\}$ in Figure 2) will have different forms when transformed to the platform frame $\{p\}$.

The region in platform space where a task-space accommodation matrix transforms to a dominant matrix in the actuator space is called the *dominant matrix kernel* (DMK) of the task-space accommodation matrix. Remembering that we can algorithmically synthesize any dominant matrix in the actuator space, we can say that a wrist can be programmed to assume a desired task-space accommodation matrix anywhere inside the DMK.

In Figure 1, we have considered a particular task space accommodation matrix and have located its DMK. The significant volume of space wherein we can locate the center of accommodation of the wrist is an indication of the power of such a wrist. This is a potential improvement over the RCC design where the center of compliance can be located at only one point in space.

References

1. Anderson, B. D. O. and S. Vongpanitlerd. *Network Analysis and Synthesis: A Modern Systems Theory Approach.* Prentice Hall, Inc. Englewood Cliffs, NJ (1973)

2. Cederbaum, I. *A Generalization of the "No-Amplification" Property of Resistive Networks.* IRE Transactions on Circuit Theory **CT-5**(224)(1958)

3. Civalleri, P. P. *Recent Trends in the Synthesis of Single-Element-Kind n-Ports.* Network and Switching Theory - A NATO Advanced Study Institute. Biorci ed. Academic Press. New York (1968)

4. Colgate, J. E. *The Control of Dynamically Interacting Systems.* Ph.D., Massachusetts Institute of Technology (1988)

5. Colgate, J. E. and N. Hogan. *An analysis of contact instability in terms of passive physical equivalents.* Proceedings of the 1989 IEEE International Conference on Robotics and Automation. IEEE Press (1989)

6. Cutkosky, M. R. and P. K. Wright. *Active Control of a Compliant Wrist in Manufacturing Tasks.* Journal of Engineering for Industry ; Transactions of the ASME **108**(February):36-43 (1986)

7. Karnopp, D. and R. Rosenberg. *System Dynamics: A Unified Approach.* John Wiley & Sons. New York (1975)

8. Paynter, H. M. *Analysis and Design of Engineering Systems.* M.I.T. Press. Cambridge, Massachusetts (1961)

9. Peshkin, M. A. *Programmed Compliance for error-corrective manipulation.* IEEE Transactions on Robotics and Automation (to be published) (1990)

10. Schimmels, J. M. and M. A. Peshkin. *Synthesis and Validation of Non-Diagonal Accommodation Matrices for Error-Corrective Assembly.* Proc. IEEE International Conference on Robotics and Automation. Cincinatti IEEE Press (1990)

11. Weinberg, L. *Network Analysis and Synthesis.* McGraw-Hill Electrical and Electronic Enginerring Series. Terman ed. McGraw-Hill Book Company, Inc. New York (1962)

12. Whitney, D. E. *Quasi-static assembly of compliantly supported rigid parts.* ASME J. Dynamic Systems, Measurement, and Control. **104**:65-77 (1982)

13. Whitney, D. E. *Historical Perspective and State of the Art in Robot Force Control.* International Journal of Robotics Research **6**(1):3-14 (1987)